1. (10) Integrate

$$\int xe^{2x}\,dx.$$

parts

$u = x \qquad dv = e^{2x}\,dx$

$du = dx \qquad v = \frac{1}{2}e^{2x}$

$$uv - \int v\,du = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\,dx$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$$

2. (10) Integrate

$$\int \frac{\cos^3(\sqrt{x})}{\sqrt{x}}\, dx.$$

u-subs: $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\, dx$

$$= \int 2 \cos^3 u \, du$$

$\cos^2 u = 1 - \sin^2 u$

$$= \int 2(1 - \sin^2 u) \cos u \, du$$

v-subs: $v = \sin u$

$dv = \cos u \, du$

$$= \int 2(1 - v^2)\, dv = 2\left(v - \tfrac{1}{3}v^3\right) + C$$

$$= 2\left(\sin u - \tfrac{1}{3}\sin^3 u\right) + C$$

$$= 2\left(\sin\sqrt{x} - \tfrac{1}{3}\sin^3\sqrt{x}\right) + C$$

3. (10) Integrate

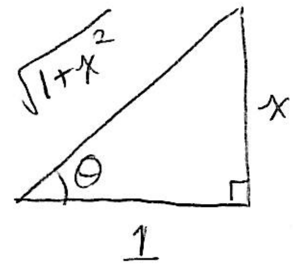
$$\int x^3\sqrt{1+x^2}\,dx.$$

Remember to put your answer in term of x.

$1+\tan^2\theta = \sec^2\theta$

let $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$



$$= \int \tan^3\theta\sqrt{1+\tan^2\theta}\ \sec^2\theta\, d\theta$$

$$= \int \tan^3\theta\ \sec^3\theta\ d\theta$$

$$= \int \tan^2\theta\ \sec^2\theta\ (\sec\theta\tan\theta\ d\theta)$$

$$= \int (1-\sec^2\theta)\sec^2\theta\ (\sec\theta\tan\theta\ d\theta)$$

$$= \int (\sec^2\theta - \sec^4\theta)(\sec\theta\tan\theta\ d\theta)$$

$$u^2 - u^4 \qquad du \qquad\qquad u = \sec\theta$$

$$= \frac{1}{3}u^3 - \frac{1}{5}u^5 + C = \frac{1}{3}\sec^3\theta - \frac{1}{5}\sec^5\theta + C$$

$$x = \tan\theta \Longrightarrow \sec\theta = \sqrt{1+x^2}$$

$$= \frac{1}{3}(1+x^2)^{3/2} - \frac{1}{5}(1+x^2)^{5/2} + C$$

4. (15) Let the origin O be the vertex of a parallelogram and let $P = (1, 0, -4)$ and $Q = (2, 2, 5)$ be the vertices adjacent to O.

(a) What is the area of the parallelogram? You need not simplify your answer.

$\overrightarrow{OP} = \langle 1, 0, -4 \rangle \qquad \overrightarrow{OQ} = \langle 2, 2, 5 \rangle$

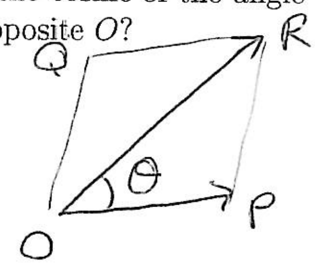
area $= |\overrightarrow{OP} \times \overrightarrow{OQ}|$

$\overrightarrow{OP} \times \overrightarrow{OQ} = (0-(-8))\vec{i} + (-8-5)\vec{j} + (2-0)\vec{k}$

$= \langle 8, -13, 2 \rangle$

area $= \sqrt{64+169+4} = \sqrt{237}$

(b) What is the cosine of the angle between the side OP and the diagonal from O to the vertex opposite O?



$\cos\theta = \dfrac{\overrightarrow{OR} \cdot \overrightarrow{OP}}{|\overrightarrow{OR}|\,|\overrightarrow{OP}|}$

$\overrightarrow{OP} = \langle 1, 0, -4 \rangle \qquad \overrightarrow{OR} = \langle 3, 2, 1 \rangle$

$|\overrightarrow{OP}| = \sqrt{17} \qquad |\overrightarrow{OR}| = \sqrt{9+4+1}$

$= \sqrt{14}$

$\overrightarrow{OR} \cdot \overrightarrow{OP} = 3 + 0 - 4 = -1$

$\cos\theta = \dfrac{-1}{\sqrt{14} \cdot \sqrt{17}}$

5. (10) Find an equation of the plane which contains the point $(1, 2, 3)$ and the line given by $x = 4 + t,\ y = 5 + 2t,\ z = 3 - t$.

point on line: $(4,5,3)$

vector parallel to line (hence in plane): $\langle 1,2,-1\rangle$

another vector in plane: $\langle 4-1, 5-2, 3-3\rangle = \langle 3,3,0\rangle$

vector orthogonal to plane: $\langle 1,2,-1\rangle \times \langle 3,3,0\rangle$

$$= (0-(-3))\vec{i} + (-3-0)\vec{j} + (3-6)\vec{k}$$

$$= \langle 3,-3,-3\rangle$$

using $(1,2,3)$ as point in plane

$$\vec{n} = \langle 3,-3,-3\rangle$$

$$\vec{r}_0 = \langle 1,2,3\rangle$$

$$\vec{n} \cdot \vec{r}_0 = 3-6-9 = -12$$

$\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{r}_0$ is

$$3x - 3y - 3z = -12$$

6. (15) Let L be the line given by $x = 1 + t,\ y = -3 + 4t,\ z = -2t$, let P_1 be the plane whose equation is $x + y + z = 4$ and let P_2 be the plane whose equation is $2x - y + 3z = 0$.

(a) Find the point of intersection of L and P_1.

$$(1+t) + (-3+4t) + (-2t) = 4$$
$$-2 + 3t = 4$$
$$3t = 6$$
$$t = 2$$

so $x = 3$

$y = -3 + 8 = 5$ $\quad (3, 5, -4)$

$z = -4$

(b) Find parametric equations of the line of intersection of P_1 and P_2.

a point on the line: If $x = 0$, solve $y + z = 4$, $-y + 3z = 0$

get $(0, 3, 1)$

(many answers possible)

Direction vector: in both planes so $\perp$ to both normals.

$$\langle 1, 1, 1 \rangle \times \langle 2, -1, 3 \rangle = (3 - (-1))\vec{i} + (2-3)\vec{j} + (-1-2)\vec{k}$$
$$= \langle 4, -1, -3 \rangle$$

parametric equations

$x = 4t$

$y = 3 - t$

$z = 1 - 3t$

7. (10) Let a curve in 3-space be given by

$$\mathbf{r}(t) = \langle \sin(3t), \cos(3t), \sqrt{7}t \rangle$$

from $t = 0$ to $t = 1$. Find the length of the curve.

$$\vec{r}\,'(t) = \langle 3\cos(3t), -3\sin(3t), \sqrt{7} \rangle$$

$$|\vec{r}\,'(t)| = \sqrt{\underbrace{9\cos^2(3t) + 9\sin^2(3t)}_{=9} + 7}$$

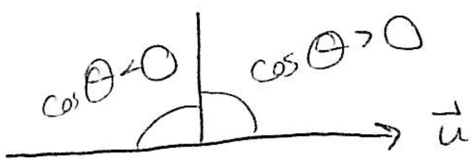
$$= \sqrt{16} = 4$$

$$\int_0^1 4\,dt = 4$$

8. (20) Multiple choice. Circle the correct response. You need not show your work. No partial credit will be given.

(a) Let $\mathbf{u}$ and $\mathbf{v}$ be non-parallel vectors and denote the scalar projection of $\mathbf{v}$ onto $\mathbf{u}$ by $\text{comp}_{\mathbf{u}}\mathbf{v}$. If $\text{comp}_{\mathbf{u}}\mathbf{v} = -2$, then the angle between $\mathbf{u}$ and $\mathbf{v}$ is

A. $< \frac{\pi}{2}$ *B.* $\frac{\pi}{2}$ *C.* $> \frac{\pi}{2}$ *D.* π *E.* None of these



(b) The parallelepiped spanned by the vectors $\langle 1,0,2\rangle$, $\langle 3,1,1\rangle$ and $\langle 1,2,5\rangle$ has volume

A. 8 *B.* 9 *C.* 10 *D.* 13 *E.* None of these

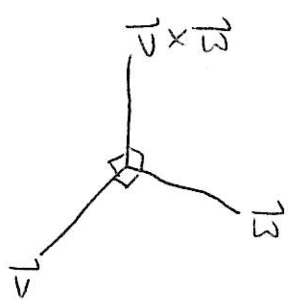
$$\langle 1,0,2\rangle \cdot (\langle 3,1,1\rangle \times \langle 1,2,5\rangle)$$

$$\begin{vmatrix} 1 & 0 & 2 \\ 3 & 1 & 1 \\ 1 & 2 & 5 \end{vmatrix} = (5-2) + (0-0) + (12-2) = 13$$

(c) If $\mathbf{v}\cdot\mathbf{w} = 0$, then $\mathbf{v}\times(\mathbf{v}\times\mathbf{w})$ is

A. Perpendicular to $\mathbf{w}$ *B.* Equal to the zero vector *C.* Parallel to $\mathbf{w}$ *D.* Not defined *E.* None of these

$$\vec{v}\cdot\vec{w} = 0 \Rightarrow \vec{v} \perp \vec{w}$$



(d) Let $\mathbf{r}(t) = \langle 2t^3, e^t, \cos(\pi t)\rangle$. Then

$$\lim_{h\to 0} \frac{\mathbf{r}(2+h) - \mathbf{r}(2)}{h} = \quad r'(2)$$

A. $\langle 16, e^2, 1\rangle$ *B*. $\langle 24, e^2, 0\rangle$ *C*. $\langle 16, e, 1\rangle$ *D*. $\langle 24, 2e, 1\rangle$ *E*. None of these

$\vec{r}\,'(t) = \langle 6t^2, e^t, -\pi \sin(\pi t)\rangle$

$\vec{r}\,'(2) = \langle 6 \cdot 4, e^2, 0\rangle$

(e) A particle moving in space has acceleration at time t given by

$$\mathbf{a}(t) = \langle 2, 6t, 12t^2\rangle$$

and has initial velocity $\mathbf{v}(0) = \langle 1, 0, 0\rangle$. Then its velocity $\mathbf{v}(t)$ at time t is
A. $\langle 3, 3, 4\rangle$ *B*. $\langle 0, 6, 24t\rangle$ *C*. $\langle 2t+1, 3t^2, 4t^3\rangle$ *D*. $\langle 2t+1+C_1, 3t^2+C_2,$ $4t^3+C_3\rangle$ *E*. None of these

$\vec{v}(t) = \langle 2t + c_1, 3t^2 + c_2, 4t^3 + c_3\rangle$

$\vec{v}(0) = \langle 1, 0, 0\rangle \Rightarrow c_1 = 1, \; c_2 = c_3 = 0$

$\vec{v}(t) = \langle 2t+1, 3t^2, 4t^3\rangle$