LECTURE OUTLINE

Trigonometric Integrals and Trigonometric Substitution

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Math 8

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Goals

Trig Review

Integration Techniques:

Trigonometric Substitution

Trigonometric Integrals

Trigonometric Substitution 1

If you see a "lonely" $\sqrt{a^2 - x^2}$, then substitute

$$x = a\sin(\theta)$$

with $\frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2}$, based on the identity

$$1 - (\sin(x))^2 = (\cos(x))^2.$$

Example 1

Find the area of enclosed by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Trigonometric Substitution 2

If you see a "lonely" $\sqrt{a^2 + x^2}$, then substitute

$$x = a\tan(\theta)$$

with $\frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2}$, based on the identity

$$1 + (\tan(x))^2 = (\sec(x))^2.$$

Example 2

Find

$$\int \frac{x}{\sqrt{x^2+9}}dx.$$

Trigonometric Integrals 2

Note we used u-substitution recalling that

$$(\cos(x))^2 + (\sin(x))^2 = 1$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x),$$

or...

Trigonometric Integrals 3

We can also us u-substitution recalling that

$$1 + (\tan(x))^2 = (\sec(x))^2$$

$$\frac{d}{dx}\tan(x) = \sec^2(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

to solve this problem.

Trigonometric Integrals 1

One should always keep in find that products of sins and $\cos$s can be "linearized", namely

$$\sin(A)\cos(B) = \frac{1}{2}(\sin(A+B) + \sin(A-B))$$

$$\sin(A)\sin(B) = \frac{1}{2}(\cos(A-B) - \cos(A+B))$$

$$\cos(A)\cos(B) = \frac{1}{2}(\cos(A-B) + \cos(A+B)).$$

Example 3

Find

$$\int (\sin(x))^2(\cos(x))^2 dx.$$

Trigonometric Substitution 3

Lastly note, if you see a "lonely" $\sqrt{x^2 - a^2}$, then substitute

$$x = a\sec(\theta)$$

with $0 \leq \theta \leq \frac{\pi}{2}$ or $\pi \leq \theta \leq \frac{3\pi}{2}$, based on the identity

$$(\tan(x))^2 = (\sec(x))^2 - 1.$$