

## MATH 8: Practice Exam I

1. Evaluate:

(a.)  $\int \tan^{-1}(x)dx$

(b.)  $\int \frac{(4x + 3)dx}{(x^2 + 1)(x + 2)}$

(c.)  $\int \frac{dx}{(9 + x^2)^{5/2}}$

2. Find the volume of the solid generated by revolving the region bounded by  $y = \frac{\ln(x)}{\sqrt{x}}$ , the  $x$ -axis and the line  $x = e$  about the  $x$ -axis.

3. Find the area bounded by the curves  $y = 1 - \sqrt{x}$  and  $y = 1 - x^2$ .

4. Test the following improper integrals for convergence or divergence.

(a.)  $\int_0^\pi \sec^2(x)dx$

(b.)  $\int_1^\infty \frac{dx}{e^x + e^{-x}}$

5. Find  $\lim_{n \rightarrow \infty} \sin(n) \ln(1 + \frac{2}{n})$  or show that the sequence diverges.

6. Determine whether the following series converge or diverge.

Explain your answer and indicate which test you use.

(a.)  $\sum_{n=1}^{\infty} \frac{n^2 - n}{n^3 + 4n + 1}$

(b.)  $\sum_{n=2}^{\infty} \frac{1}{n(1 + (\ln n)^2)}$

7. Determine whether the following series diverge, converge conditionally, or converge absolutely. Explain your answer and indicate which tests you use.

(a.)  $\sum_{n=1}^{\infty} (-1)^n \frac{n + 1}{n^2}$

(b.)  $\sum_{n=1}^{\infty} (-1)^n \frac{n^2 - 5n}{3n^2 + 1}$ .

8. Determine an upper bound on the size of the error in using  $S_9$ , the ninth partial sum, to approximate the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ .
9. Suppose  $\sum_{n=1}^{\infty} a_n$  converges, and for each  $n$ ,  $a_n \neq 0$ . What can be said about  $\sum_{n=1}^{\infty} \frac{1}{a_n}$ ?
10. A ball is dropped from a height of one meter, and on each bounce it goes two-thirds as high as before. Find the total distance traveled by the ball in coming to rest.