Homework due 27/10

1. (a.) $f(x) = x^{-2}$ $f(1) = 1$

$f'(x) = -2x^{-3}$ $f'(1) = -2$

$f''(x) = 6x^{-4}$ $f''(1) = 6$

so the 2^{nd}-degree Taylor approximation to f at 1 is

$$T_2(x) = 1 + (-2)(x-1) + \frac{6}{2!}(x-1)^2 = 1 - 2(x-1) + 3(x-1)^2$$

(b.) $f'''(x) = -24x^{-5}$. On $(0.9, 1.1)$ $x^{-5} > 0$, so

$$|f'''(x)| = 24x^{-5}.$$

This is decreasing, so for x in $(0.9, 1.1)$,

$$|f'''(x)| \leq |f'''(0.9)| = 24 \cdot 0.9^{-5}$$

So take $M = 24 \cdot 0.9^{-5}$

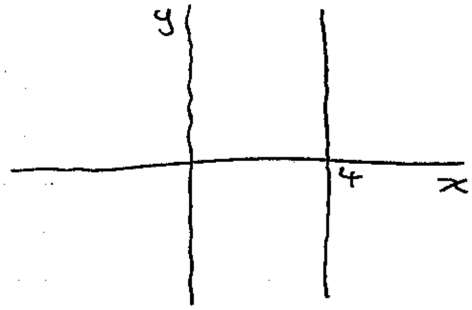
Thus by Taylor's Inequality,

$$|R_2(x)| \leq \frac{24 \cdot 0.9^{-5}}{3!}|x-1|^3.$$

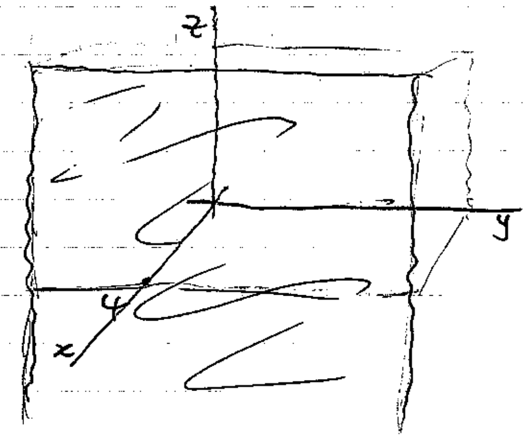
$\max |x-1| = 0.1$, so for all x in $(0.9, 1.1)$

$$|R_2(x)| \leq \frac{24}{6} \cdot 0.9^{-5} \cdot 0.1^3 \approx 0.00677$$

2. (a.) $x=4$ is a line in $\mathbb{R}^2$:



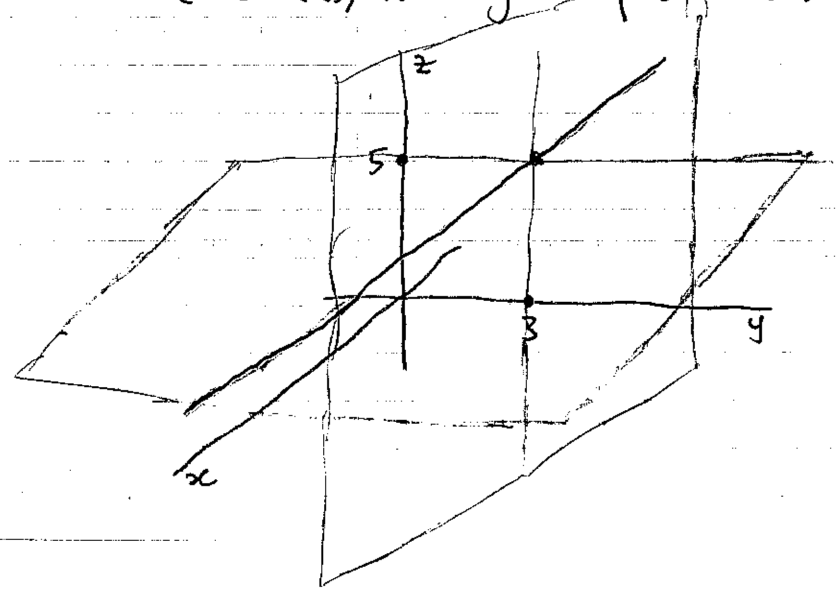
but a plane in $\mathbb{R}^3$:



(b.) $y=3$ is the plane of points parallel to the (x,z)-plane through the point $(0,0,3)$.

$z=5$ is the plane of points $(x,y,5)$ for arbitrary x,y.

The points satisfying $y=3$ and $z=5$ are a line parallel to the x-axis, namely all points $(x,3,5)$



3. $|PQ|^{\cancel{2}} = \sqrt{(1-(-2))^2+(2-4)^2+(-1+0)^2} = \sqrt{9+4+1} = \sqrt{14}$

$|PR| = \sqrt{(-1-\cancel{2}(-2))^2+(1-4)^2+(2-0)^2} = \sqrt{\cancel{9}1+9+4} = \sqrt{14}$

$|QR| = \sqrt{(-1-1)^2+(1-2)^2+(2-(-1))^2} = \sqrt{4+1+9} = \sqrt{14}$

So the triangle is equilateral.

4. (a) ~~[illegible]~~

~~[illegible]~~

$\vec{B}-\vec{A} = (7-5,\ 9-1,\ -1-3) = (2,8,-4)$

$\vec{C}-\vec{A} = (1-5,\ -15-1,\ 11-3) = (-4,-16,8) = -2\cdot(2,8,-4).$

Since B and C both lie in the direction of the vector $(2,8,-4)$ from A, A, B, C lie on a straight line.

(b.) $\vec{L}-\vec{K} = (1-0,\ 2-3,\ -2-(-4)) = (1,-1,2)$

$\vec{M}-\vec{K} = (3-0,\ 0-3,\ 1-(-4)) = (3,-3,5) = 3\cdot(1,-1,\tfrac{5}{3})$

So M and ~~L~~ lie in different directions from K, thus these three do not lie on a straight line.

5. $(x-1)^2+(y+4)^2+(z-3)^2 = 5^2$

The intersection with the (x,z)-plane is the set of points which also satisfy $y=0$, i.e.

$$(x-1)^2+4^2+(z-3)^2 = 25$$

$$(x-1)^2+(z-3)^2 = 25-4^2 = 9$$

This is a circle, centre $(1,0,3)$, radius 3.

6. $$x^2+y^2+z^2 = 4x-2y$$

$$x^2-4x \quad +y^2+2y \quad +z^2 = 0$$

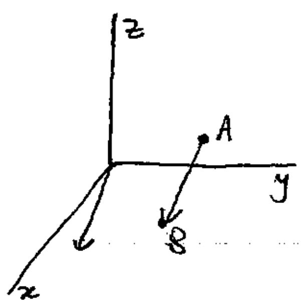
$$(x-2)^2-4 \ +(y+1)^2-1 \ +z^2 = 0$$

$$(x-2)^2+(y+1)^2+z^2 = 1+4 = 5.$$

This has centre $(2,-1,0)$ a radius $\sqrt{5}$.

7. The outer bound is the sphere of radius 5 and the inner bound is the sphere of radius 1 so the region $1 \le x^2+y^2+z^2 \le 25$ are those points between these two spheres

8. $$\underline{a} = (2-0,\ 3-3,\ -1-1) = (2,0,-2)$$



9. $$|\underline{a}| = \sqrt{(-3)^2+(-4)^2+(-1)^2} = \sqrt{26}$$

$$\underline{a}+\underline{b} = (-3+6,\ -4+2,\ -1-3) = (3,-2,-4)$$

$$\underline{a}-\underline{b} = (-3-6,\ -4-2,\ -1-(-3)) = (-9,-6,2)$$

$$2\underline{a} = (2\cdot(-3),\ 2\cdot(-4),\ 2\cdot(-1)) = (-6,-8,-2)$$

$$3\underline{a}+4\underline{b} = (3\cdot(-3)+4\cdot 6,\ 3\cdot(-4)+4\cdot 2,\ 3\cdot(-1)+4\cdot(-3)) = (15,-4,-15)$$

10. $|\underline{a}| = \sqrt{1^2+(-2)^2+1^2} = \sqrt{6}$

$\underline{a}+\underline{b} = \underline{i} + (-2+1)\underline{j} + \underline{k} + 2\underline{k} = \underline{i} - \underline{j} + 3\underline{k}$

$\underline{a}-\underline{b} = \underline{i} + (-2-1)\underline{j} + (1-2)\underline{k} = \underline{i} - 3\underline{j} - \underline{k}$

$2\underline{a} = 2\underline{i} + 2\cdot(-2)\underline{j} + 2\underline{k} = 2\underline{i} - 4\underline{j} + 2\underline{k}$

$3\underline{a}+4\underline{b} = 3\underline{i} + (3\cdot(-2)+4)\underline{j} + (3+4\cdot 2)\underline{k} = 3\underline{i} - 2\underline{j} + 15\underline{k}$

11. $|\langle 9,-5\rangle| = \sqrt{9^2+(-5)^2} = \sqrt{106}$

so $\frac{1}{\sqrt{106}}\cdot\langle 9,-5\rangle = \langle \frac{9}{\sqrt{106}}, -\frac{5}{\sqrt{106}}\rangle$ is a unit vector in the same direction as $\langle 9,-5\rangle$.

12. $|8\underline{i} - \underline{j} + 4\underline{k}| = \sqrt{8^2+(-1)^2+4^2} = \sqrt{81} = 9$

so the unit vector in this direction is $\frac{8}{9}\underline{i} - \frac{1}{9}\underline{j} + \frac{4}{9}\underline{k}$.

13. $|\langle -2,4,2\rangle| = \sqrt{(-2)^2+4^2+2^2} = \sqrt{24} = \sqrt{4}\cdot\sqrt{6} = 2\cdot\sqrt{6}$

so the unit vector in this direction is $\langle \frac{-2}{2\cdot\sqrt{6}}, \frac{4}{2\cdot\sqrt{6}}, \frac{2}{2\cdot\sqrt{6}}\rangle$

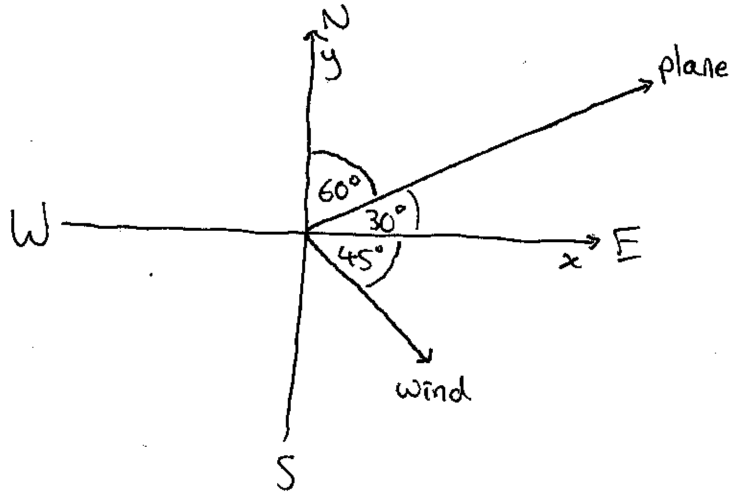
$= \langle -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\rangle.$

Thus the vector of length 6 is $\langle \frac{-6}{\sqrt{6}}, \frac{6\cdot 2}{\sqrt{6}}, \frac{6}{\sqrt{6}}\rangle = \langle -\sqrt{6}, 2\sqrt{6}, \sqrt{6}\rangle$

as can be seen by checking:

$$|\langle -\sqrt{6}, 2\sqrt{6}, \sqrt{6}\rangle| = \sqrt{(-\sqrt{6})^2 + (2\sqrt{6})^2 + (\sqrt{6})^2} = \sqrt{6+4\cdot 6+6} = \sqrt{36} = 6.$$

14.



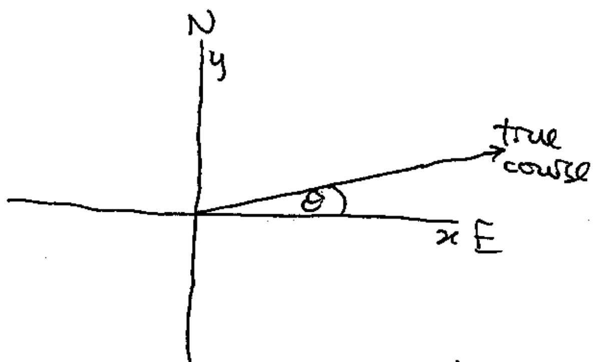
$$\underline{v}_{wind} = 50(\cos 45^\circ\, \underline{i} - \sin 45^\circ\, \underline{j}) = \frac{50}{\sqrt{2}}\underline{i} - \frac{50}{\sqrt{2}}\underline{j}$$

$$\underline{v}_{plane} = 250(\cos 30^\circ\, \underline{i} + \sin 30^\circ\, \underline{j}) = \frac{250\cdot\sqrt{3}}{2}\underline{i} + \frac{250}{2}\underline{j} = 125\sqrt{3}\,\underline{i} + 125\,\underline{j}$$

so the velocity relative to the ground is

$$\underline{v} = \underline{v}_{wind} + \underline{v}_{plane} = \left(\frac{50}{\sqrt{2}} + 125\sqrt{3}\right)\underline{i} + \left(125 - \frac{50}{\sqrt{2}}\right)\underline{j}$$

The ground speed is $|\underline{v}| = \sqrt{\left(\frac{50}{\sqrt{2}} + 125\sqrt{3}\right)^2 + \left(125 - \frac{50}{\sqrt{2}}\right)^2} \approx 267\ \text{kmh}^{-1}$



$$\theta = \tan^{-1}\left(\frac{125 - \frac{50}{\sqrt{2}}}{\frac{50}{\sqrt{2}} + 125\sqrt{3}}\right) \approx 20^\circ$$

so the course of the plane is N 70°E