LECTURE OUTLINE
Taylor and Maclaurin series Review

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Math 8

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Goals

Review Remainder Estimates

Manipulating Taylor Series

Radius of Convergence

Taylor's Inequality

Given $f(x)$ let the nth *Taylor Expansion* be $T_N(x) = \sum_{n=0}^{N} \frac{f^n(a)}{n!}(x-a)^n$, and let the Nth *Remainder* be

$R_N(x) = f(x) - T_N(x)$.

Theorem: Suppose $|f^{n+1}(x)| \leq M$ for every x in $[a, x]$ if $x > a$ (or $[x, a]$ if $x < a$), then

$$|R_N(x)| \leq M \frac{|x-a|^{N+1}}{(N+1)!}.$$

Notation I Like

We write $x = y \pm e$ to mean that x is in the interval $[y-e, y+e]$. Talyor's Inequality asserts

$$f(x) = T_N(x) \pm M\frac{|x-a|^{N+1}}{(N+1)!}.$$

(12.12: 25.) Use Taylor's Inequality to determine the number of terms of the Maclaurin series for e^x that should be be used to estimate $e^{0.1}$ to within 0.00001. (While we are at it, find a number C so that $e^{0.1} = C \pm 0.00001$.)

A Different Sort of Example

(12.Review: 56) Use series to approximate

$$\int_0^1 \sqrt{1 + x^4}dx$$

correct to two decimal places. (Due this with and without Taylor's Inequality.)

Radius of Convergence

Recall, by the ratio test, $\sum_{n=0}^{\infty} c_n(x-a)^n$ will converge if $0 < \lim_{n\to\infty} \left|\frac{c_{n+1}(x-a)}{c_n}\right| < 1$ and diverge if $1 < \lim_{n\to\infty} \left|\frac{c_{n+1}(x-a)}{c_n}\right| < \infty$.

(12.Review: 44) Find the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} x^n.$$

Manipulating Series

(12.Review: 53) Find the Maclaurin series and radius of convergence of

$$f(x) = (16 - x)^{-1/4}.$$