LECTURE OUTLINE

Taylor and Maclaurin series

Professor Leibon

Math 8

Oct. 20, 2004

Goals

Approximating functions by a Taylor Series

Taylor Remainder Estimate

Power Series Terms

Last Time We Learned: A function $f(x)$ given by power series centered at a equals $\sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$ inside its radius of convergence.

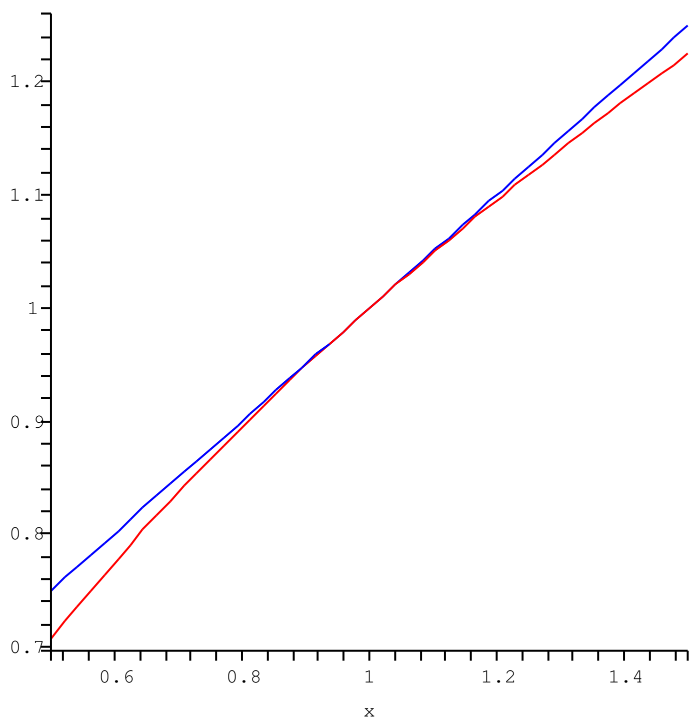
Suppose we don't know whether $f(x)$ is given by a power series, how can we interpret this $\sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$?

Tangent Line Approximation

Near a

$$f(x) \approx f(a) + f^1(a)(x-a) \equiv P_1(x,a).$$

Example: Approximate $\sqrt{1.01}$.

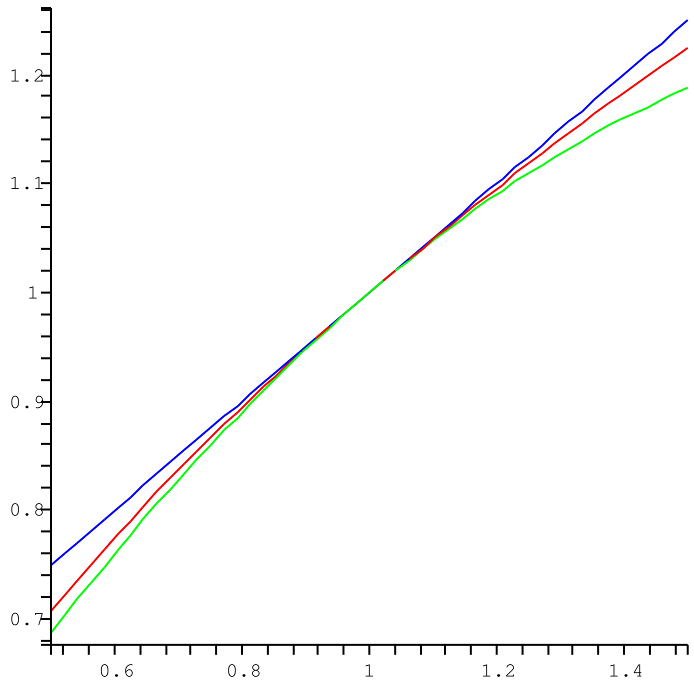


Quadratic Approximation

Even better, near a

$$f(x) \approx f(a) + f^1(a)(x-a) + \frac{1}{2}f^2(a)(x-a)^2 \equiv P_2(x,a).$$

Example: Better approximate $\sqrt{1.01}$.



Quantitative Estimate

Given $f(x)$ let the nth *Taylor Expansion* be $T_N(x) = \sum_{n=0}^{N} \frac{f^n(a)}{n!}(x-a)^n$, and let the Nth *Remainder* be

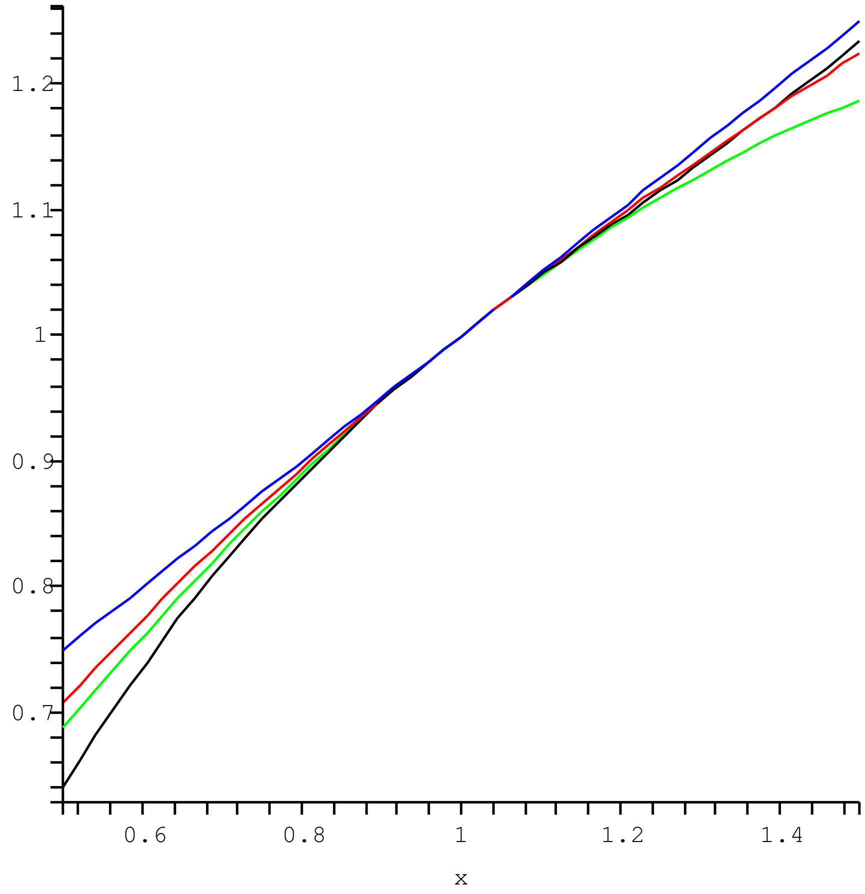
$R_N(x) = f(x) - T_N(x)$.

Theorem: Suppose $|f^{n+1}(x)| \leq M$ for every x in $[a, x]$ if $x > a$ (or $[x, a]$ if $x < a$), then

$$|R_N(x)| \leq M \frac{|x-a|^{N+1}}{(N+1)!}.$$

At least how good was our approximation of $\sqrt{1.01}$?

The Next Term



The Usual Demand

Example: How many terms of the MacClaurin series of $\sin(x)$ do you need to estimate $\sin(1)$ to with in 0.001? Compute $\sin(1)$ to with in 0.001. (This is $\sin$ of 1 radian.)