LECTURE OUTLINE
Sequences and Limits

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Math 8

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Goals

Improper Integrals

The Integral Comparison Test

Sequences

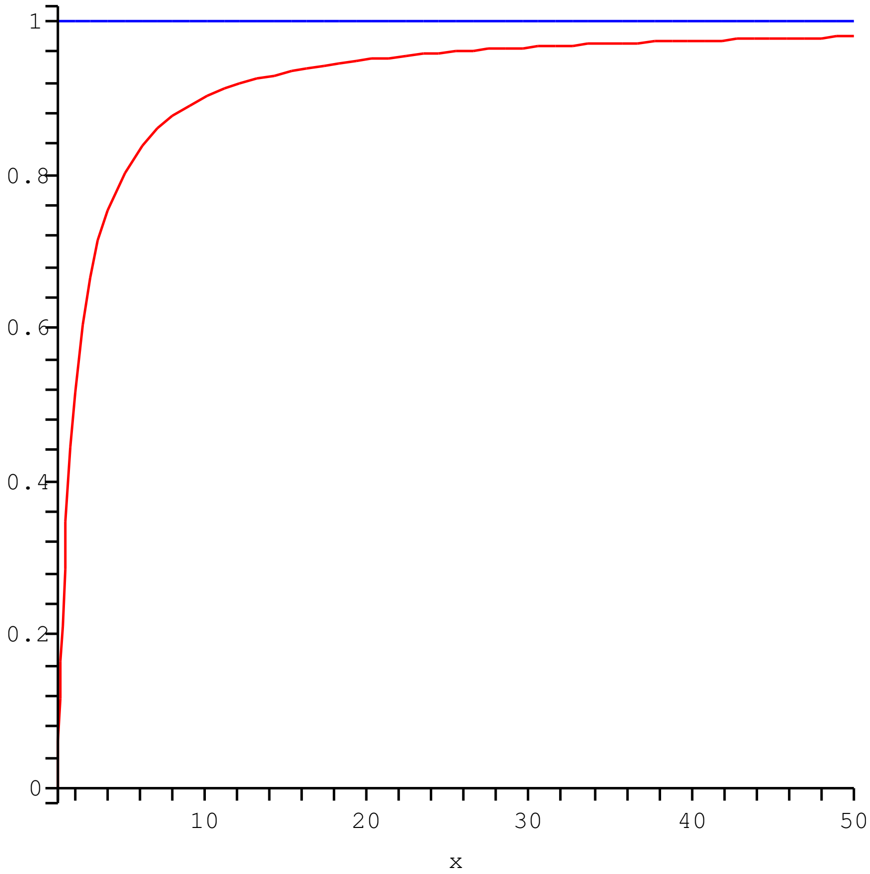
Improper Integral

If $\int_a^t f(x)dx$ exists for every number $t \geq a$, then

$$\int_a^\infty f(x)dx = \lim_{t\to\infty} \int_a^t f(x)dx$$

provided this limit exists (as a finite number). We say the integral is *convergent* if the limit exist and *divergent* otherwise.

Practice Example: $\int_1^\infty \frac{1}{x^2}dx = \lim_{t\to\infty}(1 - \frac{1}{t}) = 1.$



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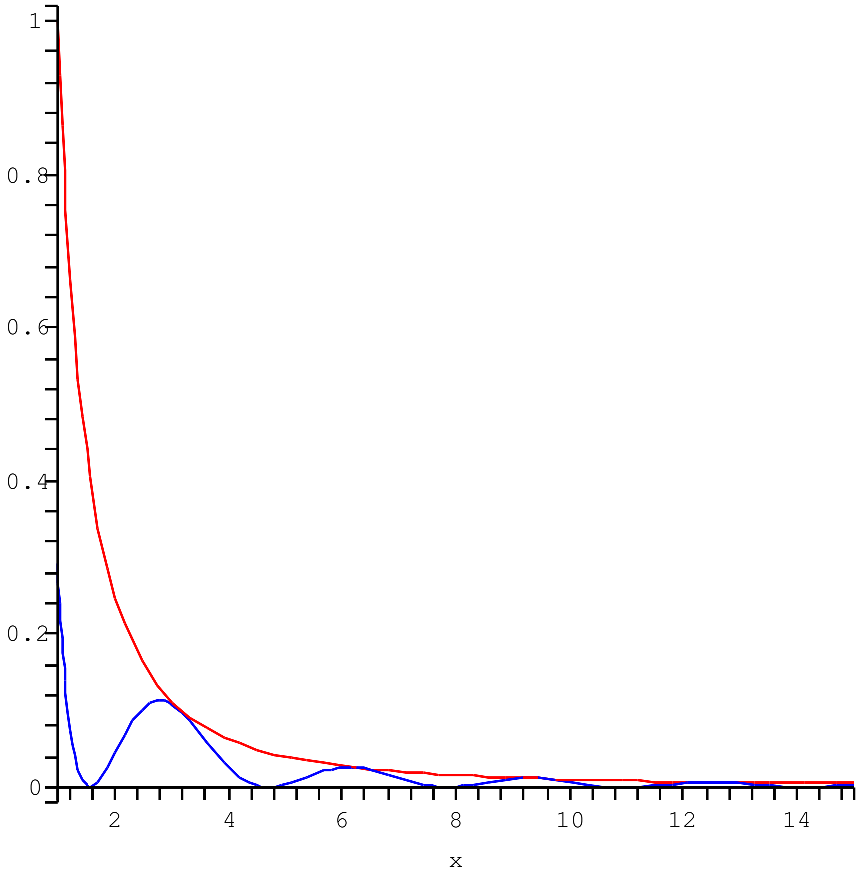
The Integral Comparison Test

Comparison Theorem: Suppose $f(x)$ and $g(x)$ are continuous functions with $f(x) \geq g(x) \geq 0$ for all $x > a$.

(a) If $\int_a^\infty f(x)dx$ is convergent, then $\int_a^\infty g(x)dx$ is convergent.

(b) If $\int_a^\infty g(x)dx$ is divergent , then $\int_a^\infty f(x)dx$ is divergent .

Example: Decide whether $\int_1^\infty \frac{(cos(x))^2}{x^2}dx$ and $\int_1^\infty \frac{3+e^{-2x}}{x}dx$ are divergent or convergent.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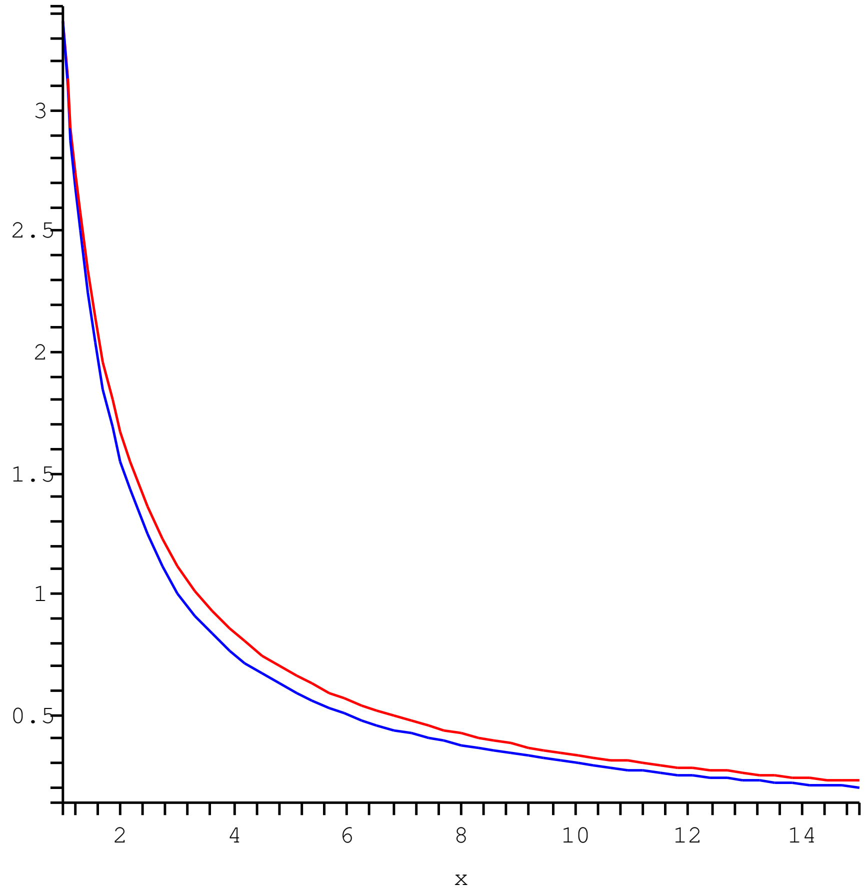
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Improper Integral

If $f(x)$ is continuous on $[a, b)$ and is discontinuous at b, then

$$\int_a^b f(x)dx = \lim_{t \to b} \int_a^t f(x)dx$$

provided this limit exists (as a finite number), and we call the integral *convergent*.

Example: Decide whether $\int_0^1 \frac{1}{\sqrt{1-x}} dx$. is divergent or convergent, and find its value if it is convergent.

A Sequence

A *sequence* is a list of numbers $a_1, a_2, a_3 \ldots, a_n \ldots$, often denoted as $\{a_1, a_2, a_3 \ldots\}$, $\{a_n\}_{n=1}^{\infty}$ or simply

$$\{a_n\}.$$

A Limit

A sequence $\{a_n\}$ has *limit* L provided **for every** $\varepsilon > 0$ **there exist** an integer N such that for every $n > N$

$$|a_n - L| < \varepsilon.$$

A Convergent Sequence

If $\{a_n\}$ has a limit L, we say $\{a_n\}$ is *convergent* and we denote this as $a_n \to L$ as $n \to \infty$ or

$$\lim_{n\to\infty} a_n = L.$$

When $\{a_n\}$ has no limit we call $\{a_n\}$ *divergent*.

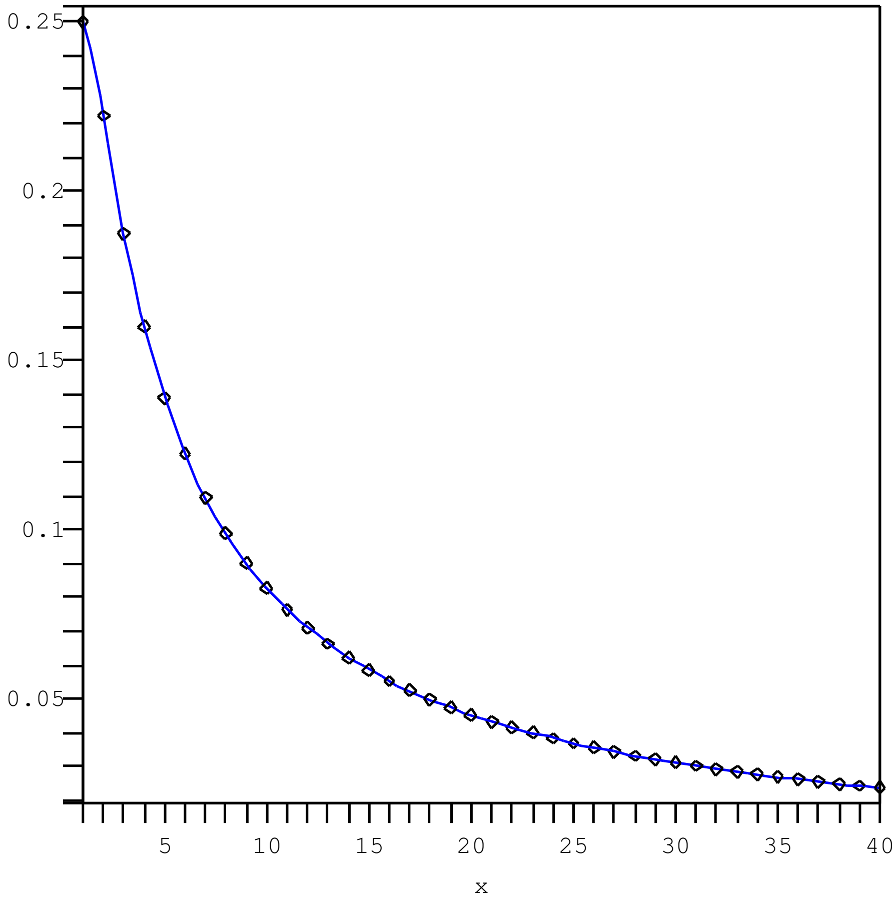
Example: Decide whether $\{(-1)^n\}$ is convergent or divergent.

Sequences Given by a Formula

If $\lim_{x\to\infty} f(x) = L$ and $a_n = f(n)$, then

$$\lim_{x\to\infty} a_n = L.$$

Example: Find the limit of $\{\frac{n}{(n+1)^2}\}$.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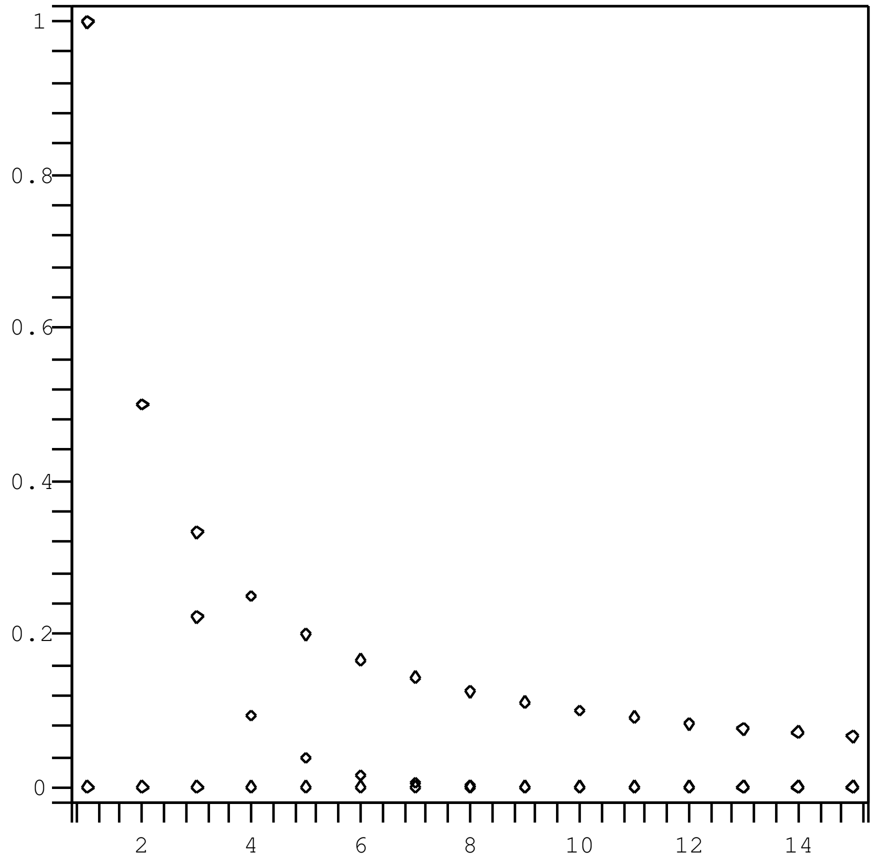
Squeeze Theorem

The Squeeze Theorem: If $a_n \leq b_n \leq c_n$ for $n > N$ and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$, then $\lim_{n\to\infty} a_n = L$.

Corollary: If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$.

Example: Find the limit of $\{\frac{n!}{n^n}\}$ and $\{\frac{(-1)^n}{n}\}$.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