LECTURE OUTLINE

Taylor Series

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Math 8

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Goals

Taylor Series

Power Series Representations of our Favorite Functions!

Power Series Terms

A function given by

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n.$$

with radius of convergence $R > 0$ satisfies

$$\frac{d^n f}{dx^n}(a) = f^n(a) = n! c_n,$$

hence

$$c_n = \frac{f^n(a)}{n!}.$$

Taylor Series

Make the following (correct!) guesses:

$$\sin(x) \text{``}=\text{''} \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

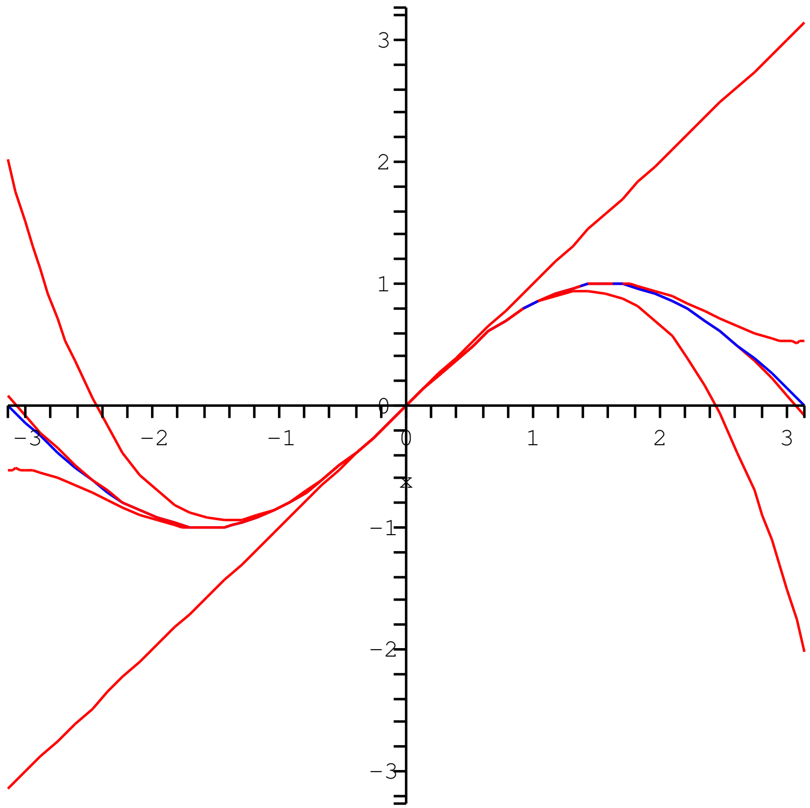
$$\cos(x) \text{``}=\text{''} \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$$

$$e^x \text{``}=\text{''} \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

and find their radii of convergence.

Ridding the Quotes in "$=$"

We need to show for our examples that at every x
$R_N(x) = f(x) - \sum_{n=0}^{N} \frac{f^n(a)}{n!}(x-a)^n$ satisfies
$\lim_{N\to\infty} R_N(x) = 0$. (Next Time: Taylor's estimate.)



SQUIDOLICIOUS!

Demonstrate (memorize!)

$$e^{ix} = \cos(x) + i\sin(x).$$

Euler's Epitaph

$$e^{i\pi} + 1 = 0$$

Formulas We've Used Again and Again and Again

Demonstrate (do not memorize!)

$$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(x + y) = 2\cos(x)\sin(x)$$