

LECTURE OUTLINE

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Power Series

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Math 8

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Goals

Power Series

Radius of Convergence

Differentiation and Integration of
Power Series

Power Series

A function given by

$$f(x) = \sum_{n=0}^{\infty} c_n (x - a)^n.$$

is called a power series. Its domain is the set of x where this series converges.

Ex: Where does $\sum_{n=1}^{\infty} x^n$ converge, and simplify this function where it converges.

Radius of Convergence

$f(x) = \sum_{n=0}^{\infty} c_n(x - a)^n$ either

(a) converges only at a

(b) converges for all x

(c) there is an $R > 0$ called the *radius of convergence* such that

$f(x)$ converges for all x such that $a - R < x < a + R$ and

diverges for all $a + R < x$ and $x < a - R$.

Ex: Find the radius of convergence of

$$\sum_{n=1}^{\infty} 2^n (x - 7)^n.$$

Differentiation and Integration

If $f(x) = \sum_{n=0}^{\infty} c_n(x - a)^n$ has a radius of convergence R , then

$$\frac{df}{dx} = \sum_{n=1}^{\infty} n c_n (x - a)^{n-1}, \text{ and}$$

$$\int f dx = \sum_{n=0}^{\infty} \frac{c_n}{n+1} (x - a)^{n+1} + C$$

in $(a - R, a + R)$, and each of these power series has radius of convergence R .

Examples

Ex: Find a power series expansion of $\log(1 - x)$ about $x = 0$ and its radius of convergence.

Ex: Find a power series expansion of $\arctan(x)$ about $x = 0$ and find its radius of convergence.

Exciting Examples

Find the radius of convergence of the power series expansion of

$$\frac{1}{1+x}$$

about $a = 0$, $a = 1$, and $a = C$.

Explore the same question for

$$\frac{1}{1+x^2}.$$