Homework due Nov. 5

(1) Find parametric & symmetric equations for the line of intersection of the planes $x+y+z=1$ & $x+z=0$.

First, we need to find a point on the line of intersection. We'll let $x=0$ and find a point that satisfies both equations.
$x=0 \Rightarrow z=0$ from eq'n 2. $x=0$ & $z=0 \Rightarrow y=1$ from eq'n 1. So the point we're seeking is $(0,1,0)$. Next, we need to find the direction of the line. We will find the cross product of the normal vectors of the planes.

$$v = n_1 \times n_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = (1-0)\vec{i} + (1-1)\vec{j} + (0-1)\vec{k} = \langle 1,0,-1 \rangle$$

So $v = \langle 1,0,-1 \rangle$ & $P_0 = (0,1,0)$ and we have parametric equations: $x=t \quad y=1 \quad z=-t$ & we have symmetric equations: $x=-z, \ y=1$

(2) (a) Find parametric equations for the line through $(5,1,0)$ that is perpendicular to the plane $2x-y+z=1$. (b) In what points does this line intersect the coordinate planes?

(a) The normal vector of the plane $2x-y+z=1$ is $\langle 2,-1,1 \rangle$ and this vector is perpendicular to the plane, so we shall use it as the direction of our line. $v = \langle 2,-1,1 \rangle$ & $P_0 = (5,1,0)$ Our parametric equations are $x = 2t+5 \quad y = -t+1 \quad z = t$

(b) The line intersects the x-y plane when $z=0$. So $t=0 \Rightarrow x=5$ & $y=1$.
" " " " y-z plane " $x=0$. So $t=-5/2 \Rightarrow y = 7/2$ & $z = -5/2$
" " " " x-z plane " $y=0$. So $t=1 \Rightarrow x=7$ & $z=1$

So the point of intersection of the line with the x-y plane is $(5,1,0)$
" " " " " " " " " y-z " " $(0, 7/2, -5/2)$
" " " " " " " " " x-z " " $(7,0,1)$

(3) & (4) Determine whether the lines L_1 & L_2 are parallel, skew, or intersecting. If the intersect, find the point of intersection.

(3) $L_1: x=-6t \quad y=1+9t \quad z=-3t \qquad L_2: x=1+2s \quad y=4-3s \quad z=s$

Consider the direction vectors: $\vec{v}_1=\langle -6,9,-3\rangle \quad \vec{v}_2=\langle 2,-3,1\rangle$

Note that $\vec{V}_1=-3\vec{V}_2 \Rightarrow L_1$ & L_2 are parallel.

(4) $L_1: x=1+2t \quad y=3t \quad z=2-t \qquad L_2: x=-1+s \quad y=4+s \quad z=1+3s$

Consider the direction vectors: $\vec{V}_1=\langle 2,3,-1\rangle \quad \vec{V}_2=\langle 1,1,3\rangle$

These vectors aren't parallel, so the lines aren't parallel

Do the lines intersect? If so then there is 1 t & 1 s that give the same point from the parametric equations.

So $1+2t=-1+s$
$3t=4+s$
$2-t=1+3s$

We'll use the first two equations to find an s & t and see if they work in the third.

From eq'n 1, $s=2+2t$. Substituting in the third,

$3t=4+(2+2t) \Rightarrow t=6 \Rightarrow s=14. \qquad 2-(6)\overset{?}{=}1+3(14)$

Since $-4\neq 43$, the s & t we found don't work $\Rightarrow$ the lines don't intersect. Since they are also not parallel, the lines must be skew.

(5) Find an equation of the plane through the point $(-2,8,10)$ and perpendicular to the line $x=1+t,\ y=2t,\ z=4-3t$.

Since the line is perpendicular to the plane, we can use its direction vector as the normal vector of the plane. $\vec{n}=\langle 1,2,-3\rangle$

So, $1(x-(-2))+2(y-8)+-3(z-10)=x+2+2y-16-3z+30=0$

$\Rightarrow x+2y-3z=-16$

⑥-⑧ Find an equation of the plane

⑥ The plane through the point $(^-1, 6, ^-5)$ & parallel to the plane $x+y+z+2=0$.

Since the two planes are parallel, their normal vectors are the same. $\vec{n} = \langle 1, 1, 1 \rangle$ So $(x-^-1)+(y-6)+(z-^-5)=0$

$\Rightarrow x+1+y-6+z+5=0 \Rightarrow x+y+z=0$

⑦ The plane through the points $A=(0,1,1)$ $B=(1,0,1)$ & $C=(1,1,0)$

We need to find the normal vector to the plane. So take the cross product of two vectors in the plane. $\vec{V}_1 = (\overrightarrow{A-B}) = \langle -1, 1, 0 \rangle$

$\vec{V}_2 = (\overrightarrow{B-C}) = \langle 0, 1, -1 \rangle$ $\quad \vec{n} = \vec{V}_1 \times \vec{V}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = (-1-0)\vec{i} + (0-1)\vec{j} + (-1-0)\vec{k}$

so $\vec{n} = \langle -1, -1, -1 \rangle$ Choose $P_0 = (0,1,1)$

So $-1(x-0)-1(y-1)-1(z-1)=0 \Rightarrow -x+0-y+1-z+1=0$

$\Rightarrow x+y+z=2$

⑧ The plane that passes through the point $(1,2,3)$ and contains the line $x=3t$ $y=1+t$ $z=2-t$

We need two non-parallel vectors to cross and find the normal vector of the plane. We have one in the direction of the line. $\vec{V}_1 = \langle 3, 1, -1 \rangle$

Since $(1,2,3)$ does not lie on the line, we can find another vector by connecting it with a point on the line. Let $t=0$ to find said point, $(0,1,2)$. So $\vec{V}_2 = \langle 1-0, 2-1, 3-2 \rangle = \langle 1, 1, 1 \rangle$

$\vec{n} = \vec{V}_1 \times \vec{V}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = (1-^-1)\vec{i} + (-1-3)\vec{j} + (3-1)\vec{k}$

$= \langle 2, -4, 2 \rangle$

So $2(x-1)+-4(y-2)+2(z-3)=0 \Rightarrow 2x-2-4y+8+2z-6=0$

$\Rightarrow 2x-4y+2z=0$

(9) Find the point at which the line intersects the given plane.

$x=1+2t$, $y=4t$ $z=2-3t$; $x+2y-z+1=0$

We substitute the parametric equations into the equation for the plane & solve for t.

$(1+2t)+2(4t)-(2-3t)+1=0 \Rightarrow 1+2t+8t-2+6t+1=0$

$\Rightarrow 16t+0=0 \Rightarrow t=0$ Substituting this back into the parametric equations, we have $x=1+2(0)$ $y=4(0)$ $z=2-3(0)$ or $(1,0,2)$

(10) & (11) Determine whether these planes are parallel, perpendicular, or neither. If neither, find the angle between them.

(10) $2z=4y-x$, $3x-12y+6z=1$

First, look at the normal vectors of the planes.

$\vec{n}_1=\langle -1,4,-2\rangle$ $\vec{n}_2=\langle 3,-12,6\rangle$

Since $\vec{n}_2=-3\vec{n}_1$, the normal vectors are parallel & hence the planes are parallel

(11) $2x-3y+4z=5$, $x+6y+4z=3$

Again, look at the normal vectors of the planes.

$\vec{n}_1=\langle 2,-3,4\rangle$ $\vec{n}_2=\langle 1,6,4\rangle$

These vectors are not parallel, so the planes are not parallel. What about perpendicular? If so, then $\vec{n}_1\cdot\vec{n}_2=0$.

$\vec{n}_1\cdot\vec{n}_2 = 2\cdot1+-3\cdot6+4\cdot4=2-18+16=0$ so $\vec{n}_1\perp\vec{n}_2$ which means the planes are also perpendicular.