LECTURE OUTLINE

Space Curves

Professor Leibon

Math 8

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Goals

Vector functions

space curves

derivatives

integrals

Length

Position

We describe a particle's position at time t via a *vector valued function*

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k} =< x(t), y(t), z(t) >,$$

with $x(t)$, $y(t)$, and $z(t)$ differentiable functions of t for (sometimes we specify for t is in a given interval $[a, b]$).

Example: Describe the *space curve* traced out by a particle following $\vec{u}(t) =< 2t + 3, 4t, -t + 7 >$.

Another Space Curve

Describe the *space curve* traced out by a particle following

$$\vec{v}(t) =< \cos(t), \sin(t), t > .$$

This curve is called a helix.

The Velocity Vector

$\vec{r}(t)$'s instantaneous change at time t, *velocity*, equals

$$\lim_{\Delta t \to 0} \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t} = \frac{d}{dt}\vec{r}(t),$$

2. Find the velocity at each time of particle traveling along a helix. Watch is the direction of this velocity, this is usually denoted $\hat{T}(t)$ and called the *unit tangent vector*.

Integration

For $t \geq a$

$$\vec{r}(t) = \int_a^t \frac{d}{dt}\vec{r}(t)dt + \vec{r}(a)$$

where we integrate each component.
3. Describe the curves that share our helix's velocity vector at each time.

Speed and Path Length

$\vec{r}(t)$'s *Speed* is given by $|\frac{d}{dt}\vec{r}(t)|$. while

$$L = \int_a^b |\frac{d}{dt}\vec{r}(t)|dt$$

is the distance traveled while t went from a to b, in other words the curve's *length*.

4. Find the length of out helix for t in $[0, B]$.

Differentiation Rules

1. $\frac{d}{dt}[\vec{u}(t) + \vec{v}(t)] = \frac{d\vec{u}}{dt}(t) + \frac{d\vec{v}}{dt}(t)$
2. $\frac{d}{dt}[c\vec{u}(t)] = c\frac{d\vec{u}}{dt}(t)$
3. $\frac{d}{dt}[f(t)\vec{u}(t)] = f(t)\frac{d\vec{u}}{dt}(t) + \frac{df}{dt}(t)\vec{u}$
4. $\frac{d}{dt}[\vec{u}(t) \cdot \vec{v}(t)] = \frac{d\vec{u}}{dt}(t) \cdot \vec{v}(t) + \vec{u}(t) \cdot \frac{d\vec{v}}{dt}(t)$
5. $\frac{d}{dt}[\vec{u}(t) \times \vec{v}(t)] = \frac{d\vec{u}}{dt}(t) \times \vec{v}(t) + \vec{u}(t) \times \frac{d\vec{v}}{dt}(t)$
6. $\frac{d}{dt}[\vec{u}(f(t))] = \frac{d\vec{u}}{dt}(t)\frac{df}{dt}(t)$

Here We Go!!!

Let $\vec{v}(t) =< \cos(t), \sin(t), t >$ and let $\hat{T}(t)$ be its unit tangent vector. Show $\hat{T} \cdot \frac{d}{dt}\hat{T} = 0$ Explain why this is always true.

Find $\frac{d}{dt}\hat{T}(t)$'s direction vector for our helix, and call it $\hat{N}(t)$ the *curve's normal vector*.

Let $\hat{B}(t) = \hat{T}(t) \times \hat{N}(t)$, this is usually called the curve's *binormal vector*. Compute $\hat{B}$ and $\frac{d}{dt}\hat{B}$ for our helix. Why must $\frac{d}{dt}\hat{B}$ be a multiple of $\hat{N}$?

Show that $\frac{d}{dt}\hat{N}(t) = -a(t)\hat{T} + b(t)\hat{B}$, for our helix. Why is this always true?