LECTURE OUTLINE

Extreme Values

Professor Leibon

Math 8

Nov. 29, 2004

Extreme Values

Extreme Value Theorem: If f is a continuous on a closed, bounded set D in $\mathbf{R}^2$, then f attains an absolute maximum value $f(x_1, y_1)$ and an absolute minimum value $f(x_2, y_2)$ at some points (x_1, y_1) and (x_2, y_2) in D.

To find the absolute maximum and minimum values of a continuous function on a closed, bounded set D:

1. Find the value of f at each of the critical points of f in D.
2. Find the extreme values of f on the boundary of D.
3. The largest of the values from steps 1 and 2 is the absolute maximum while the smallest is the absolute minimum.

Example

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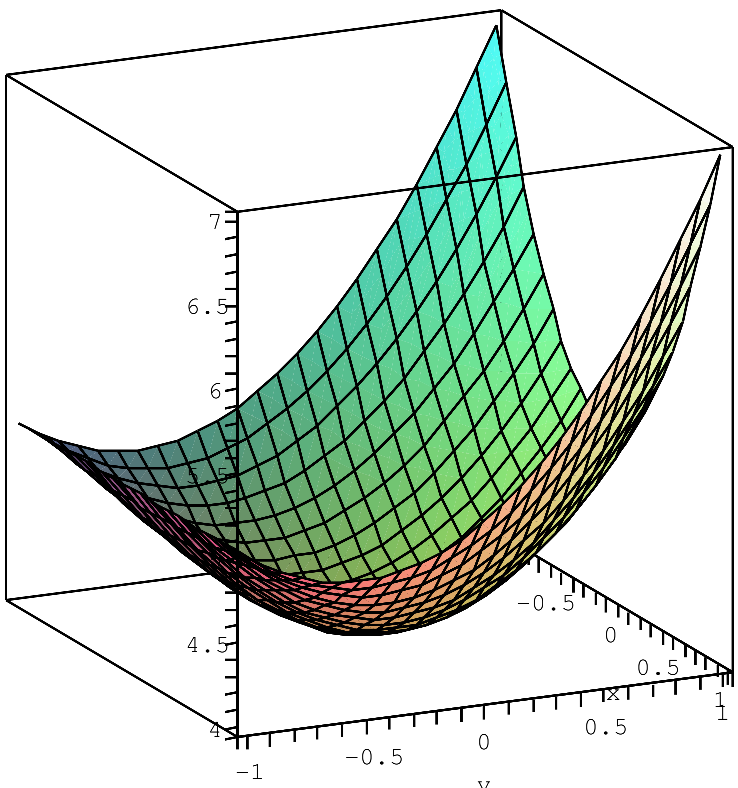

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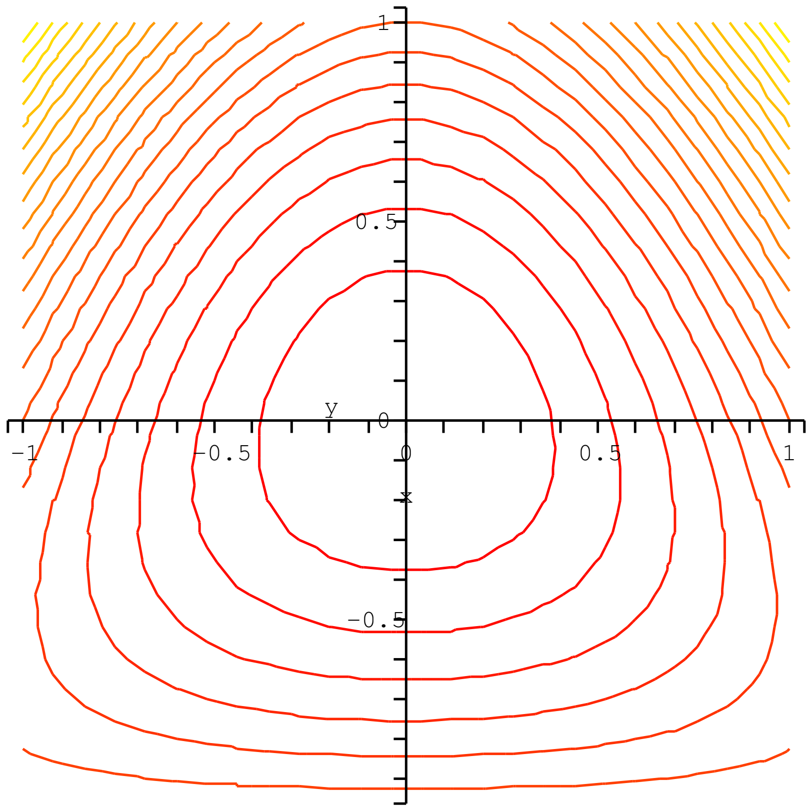

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Example 2: Find the dimensions of the largest volume rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $x + 2y + 3z = 6$.

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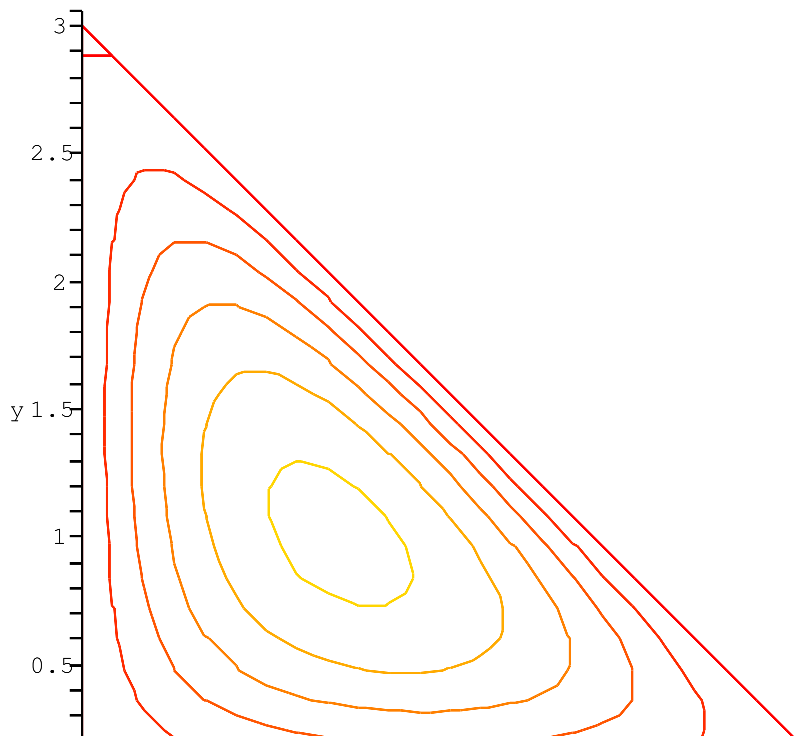

Lagrange Multipliers

The Method of Lagrange Multipliers: To find the maximum and minimum values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$ (assuming the extreme value exist and $\nabla g \neq 0$ on the surface $g(x, y, z) = k$):

(a) Find all x, y, z, and λ such that $\nabla f(x, y, z) = \lambda \nabla g(x, y, z)$ and $g(x, y, z) = k$.

(b) Evaluate f at all the points (x, y, z) that result from step (a). The largest of these is the maximum value of f; the smallest is the minimum value of f.

Example

Example 2 (revisited): Use the Method of Lagrange Multipliers to find the dimensions of the largest volume rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $x + 2y + 3z = 6$.

Example

Example 3: Find the dimensions of the largest volume rectangular box without a lid that can be made from $12m^2$ of cardboard.