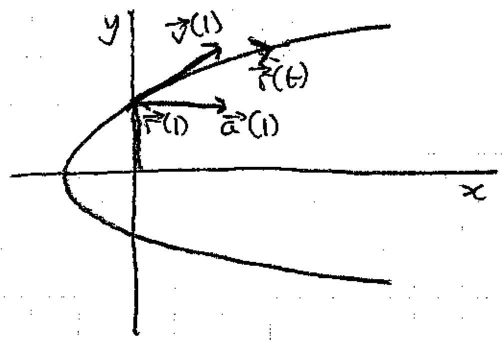
Homework due 11/10

1. $\vec{r}(t) = \langle t^2 - 1, t \rangle$

$\vec{v}(t) = \vec{r}\,'(t) = \langle 2t, 1 \rangle$ is the velocity at time t.

$\vec{a}(t) = \vec{v}\,'(t) = \langle 2, 0 \rangle$ is the acceleration at time t.

$v(t) = |\vec{v}(t)| = \sqrt{(2t)^2 + 1^2} = \sqrt{4t^2+1}$ is the speed at time t.



2. $\vec{r}(t) = \langle \sqrt{2}\,t, e^t, e^{-t} \rangle$

$\vec{v}(t) = \langle \sqrt{2}, e^t, -e^{-t} \rangle$

$\vec{a}(t) = \langle 0, e^t, e^{-t} \rangle$

$v(t) = |\vec{v}(t)| = \sqrt{(\sqrt{2})^2 + (e^t)^2 + (-e^{-t})^2} = \sqrt{2 + e^{2t} + e^{-2t}}$

$= \sqrt{\dfrac{2e^{2t} + e^{4t} + 1}{e^{2t}}} = \dfrac{\sqrt{(e^{2t}+1)^2}}{\sqrt{e^{2t}}} = \dfrac{e^{2t}+1}{e^t} = e^t + e^{-t}$

3. $\vec{r}(t) = \langle t\sin t, t\cos t, t^2 \rangle$

$\vec{v}(t) = \langle \sin t + t\cos t, \cos t - t\sin t, 2t \rangle$

$\vec{a}(t) = \langle \cos t + \cos t - t\sin t, -\sin t - \sin t - t\cos t, 2 \rangle$

$= \langle 2\cos t - t\sin t, -2\sin t - t\cos t, 2 \rangle$

$v(t) = \sqrt{(\sin t + t\cos t)^2 + (\cos t - t\sin t)^2 + (2t)^2}$

$= \sqrt{(\sin^2 t + \cancel{2t\sin t\cos t} + t^2\cos^2 t) + (\cos^2 t - \cancel{2t\cos t\sin t} + t^2\sin^2 t) + 4t^2}$

$$= \sqrt{(\sin^2 t + \cos^2 t) + t^2(\cos^2 t + \sin^2 t) + 4t^2}$$

$$= \sqrt{1 + 5t^2}$$

4. $\vec{a}(t) = \vec{k}$ so $\vec{v}(t) = \int \vec{a}(t)dt = \int \vec{k}\, dt = t\vec{k} + \vec{c_1}$

But $\vec{i} - \vec{j} = \vec{v}(0) = 0 \cdot \vec{k} + \vec{c_1} = \vec{c_1}$

so $\vec{v}(t) = \vec{i} - \vec{j} + t\vec{k}$.

Then $\vec{r}(t) = \int \vec{v}(t)dt = \int (\vec{i} - \vec{j} + t\vec{k})\, dt = t\vec{i} - t\vec{j} + \frac{1}{2}t^2\vec{k} + \vec{c_2}$

but $\vec{0} = \vec{r}(0) = \vec{c_2}$ so $\vec{r}(t) = t\vec{i} - t\vec{j} + \frac{1}{2}t^2\vec{k}$.

5. $\vec{r}(t) = \langle t^2, 5t, t^2 - 16t \rangle$

$\vec{v}(t) = \vec{r}\,'(t) = \langle 2t, 5, 2t - 16 \rangle$

$v(t) = |\vec{v}(t)| = \sqrt{(2t)^2 + 5^2 + (2t-16)^2} = \sqrt{4t^2 + 25 + 4t^2 - 64t + 256}$

$= \sqrt{8t^2 - 64t + 281}$ is the speed of the particle

$\therefore\ v'(t) = \frac{1}{2}(16t - 64)(8t^2 - 64t + 281)^{-1/2}$

$\therefore\ v'(t) = 0$ means that $16t - 64 = 0$

i.e. $t = 4$.

This is the minimum of $v(t)$ because, since $16t - 64$ is increasing near $t=4$, $v'(t) < 0$ for $t < 4$ and $v'(t) > 0$ for $t > 4$.

So the speed is a minimum at $t=4$.

6. (a.) $f(40,15)=25$.

This means that if the wind blows for 15 hours at 40 knots then the wave height will be 25 feet.

(b.) $h=f(30,t)$ gives the wave height in terms of the time t if the wind speed is constant at 30 knots.

If we look at the row of the table corresponding to $v=30$:

Duration

$v \backslash t$	5	10	15	20	30	40	50
30	9	13	16	17	18	19	19

we see the wave height is increasing and seems to be asymptoting to near 19 feet.

(c.) $h=f(v,30)$ gives the wave height after wind speed has been constant at ~~30~~ v knots for 30 hours.

If we look at the column for $t=30$, we see the wave height increases ever-more rapidly as the windspeed is increased.

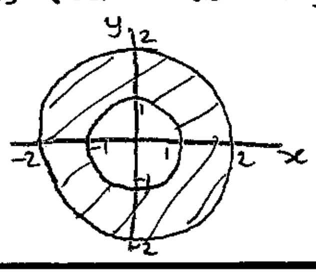
7. $\sqrt{x^2+y^2-1}$ is valid if $x^2+y^2-1 \geq 0$

i.e. for (x,y) not in the unit disk $x^2+y^2 \geq 1$

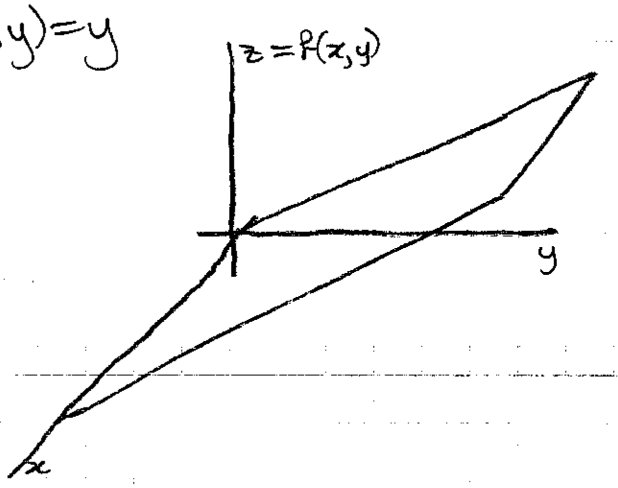
$\ln(4-x^2-y^2)$ is valid if $4-x^2-y^2 > 0$

i.e. for (x,y) in the disk $x^2+y^2 < 4$, of radius 2

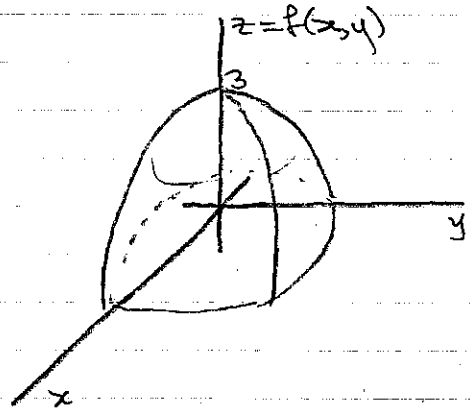
So f is valid for (x,y) between the circles of radii 1 and 2.



8. $f(x,y)=y$



9. $f(x,y)=3-x^2-y^2$



This is a circular parabaloid.

10. (a.) $f(x,y)=|x|+|y|$ is either V or VI because of the jagged lines on the x- and y- axes.
But in V the function is 0 everywhere on the axes.
This is not true for f, so f is VI.

(b.) $f(x,y)=|xy|$. f is similar to (a.), but this time
$f(x,0)=f(0,y)=0$ for all x,y, i.e. f is 0 on the axes.
So f is V.

(c.) Consider (x,y) on the circle $x^2+y^2=r^2$.
Then $f(x,y)=\frac{1}{1+r^2}$.

Thus f is symmetric about the origin and decreases as points get further from the origin (as r increases),

so f is I.

(d) The traces in the axes are

$$f(x,0) = x^4 \quad \text{in the } x\text{-axis}$$

$$f(0,y) = y^4 \quad \text{in the } y\text{-axis.}$$

so f could be II or IV.

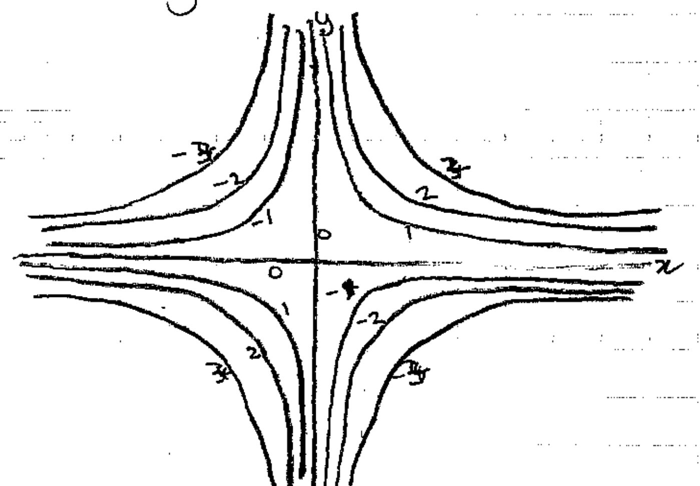
The zeros of f are at $x = \pm y$, so f is IV.

11. $f(-3,3)$ lies between the level curves with values 50 and 60, and is nearer to 60. Let's guess $f(-3,3) \approx 57$.

$(3,-2)$ lies halfway between the curves with values 30 and 40, so let's guess $f(3,-2) \approx 35$.

The graph increases as we approach the origin, most steeply from the negative x-axis.

12.



If $xy=1$ then $y=\frac{1}{x}$

" $xy=2$ " $y=\frac{2}{x}$

etc.