1. Find the maximum and minimum of $f(x, y) = x^2 + 2x + y^2$ on the disk $x^2 + y^2 \leq 4$.

2. Find the maximum and minimum of $T(x, y, z) = xyz$ subject to the constraint $x^2 + y^2 + 4z^2 = 12$. Note: It is easy to see that all absolute extrema occur when none of x, y, z are zero, so you may assume that fact in your solution.

3. Let $f(x, y) = x^3y^4$.

   (a) Find $\nabla f(1, 1)$.

   (b) Find an equation of the tangent plane to the graph of f at the point $(1, 1, 1)$.

   (c) Find the maximum rate of increase of f at $(1, 1)$, and the direction in which it occurs.

   (d) A unit vector $\mathbf{u}$ makes an angle of $\pi/3$ with $\nabla f(1, 1)$. Find the directional derivative $D_{\mathbf{u}}f(1, 1)$. Hint: you don't really need to know $\mathbf{u}$ to answer this question.

   (e) Determine an equation for the tangent line to the level curve of f at the point $(1, 1)$.

4. Consider the following series. If they converge, determine their value; if they diverge, briefly say why.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}$

   Converges to ____________________________________,

   or

   Diverges because ____________________________________

   (b) $\displaystyle\sum_{n=0}^{\infty} 10^{10} \left(\frac{2}{3}\right)^n$

   Converges to ____________________________________,

   or

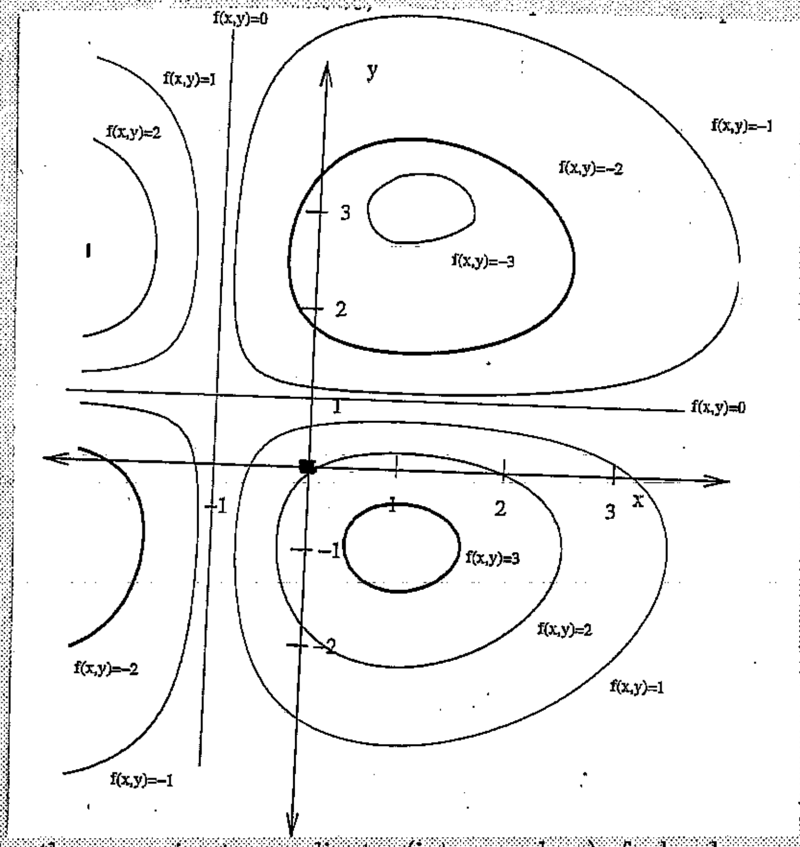
Diverges because ______________________________

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n!}$

Converges to ______________________________,
or

Diverges because ______________________________

5. Below is drawn a contour map (the level curves) for a twice differentiable function $f(x, y)$. Use this map to answer the questions which follow:



(a) Give the approximate coordinates (integer values) of a local maximum point of f.

(b) Give the approximate coordinates (integer values) of a saddle point of f.

(c) Consider the point $(0,0)$ marked on the diagram by a ■. You may assume that none of the partial derivatives of f are zero at this point. Indicate whether the following partial derivatives are positive or negative:

$f_x(0,0)$ is ______________________________

$f_y(0,0)$ is ______________________________

6. Suppose a twice differentiable function f has a critical point at (x_0, y_0). In each of the following, information about the second order partials is given. In each case, classify the critical point as a local maximum, local minimum, or saddle point, or else explain why the second derivative test fails.

   (a) $f_{xx}(x_0, y_0) = 2, f_{yy}(x_0, y_0) = 6, f_{xy}(x_0, y_0) = 2.$

   (b) $f_{xx}(x_0, y_0) = 2, f_{yy}(x_0, y_0) = 8, f_{xy}(x_0, y_0) = 4.$

   (c) $f_{xx}(x_0, y_0) = 2, f_{yy}(x_0, y_0) = 6, f_{xy}(x_0, y_0) = 5.$

7. Given a function $z = f(u)$ where $u = g(x, y)$, $x = h(s, t)$, and $y = k(s, t)$, use the Chain Rule to write an expression for $\frac{\partial z}{\partial t}$ in terms of the partial derivatives of the other functions.

8. **Multiple Choice** Circle the correct response.

   A. What is the Maclaurin series for $\frac{1}{1+x^2}$?

   a. $\sum_{n=0}^{\infty} (-1)^n x^{2n}$

   b. $\sum_{n=0}^{\infty} x^{2n}$

c. $\sum_{n=0}^{\infty}(-1)^n(2n)x^{2n-1}$

d. $\sum_{n=1}^{\infty}(-1)^n\frac{x^{2n-1}}{2n}$

e. none of the above

B. If the motion of a particle is given by a vector-valued function $\mathbf{r}(t)$ defined for $a \le t \le b$, then the integral of the speed of $\mathbf{r}(t)$ from $t = a$ to $t = b$ equals

a. the acceleration

b. the distance from $\mathbf{r}(a)$ to $\mathbf{r}(b)$

c. the velocity

d. the distance the particle travels going from $\mathbf{r}(a)$ to $\mathbf{r}(b)$

e. none of the above

C . Which of the following vectors is orthogonal to the plane containing the parallel lines $\mathbf{r}(t) = \langle 1,1,1\rangle + t\langle 2,1,4\rangle$, and $\mathbf{r}(t) = \langle 2,3,4\rangle + t\langle 2,1,4\rangle$.

a. $\langle 1,1,1\rangle \times \langle 2,3,4\rangle$

b. $\langle 1,1,1\rangle \times \langle 2,1,4\rangle$

c. $\langle 2,1,4\rangle$

d. $\langle 1,2,3\rangle \times \langle 2,1,4\rangle$

e. $\langle 1,2,3\rangle$

9. Find
$$\int \frac{1}{x^2\sqrt{x^2+9}}dx.$$

10. Determine whether the planes given by $x + 4y - 3z = 1$ and $-3x + 6y + 7z = 3$ are parallel, perpendicular, or neither. If neither, find the angle between them.