Set Theory Week 5 due Friday Feb. 12th

* **1)** Consider $FS(\beta,\alpha) = \{ f : \beta \to \alpha \,/\, |\{\gamma \in \beta \,/\, f(\gamma) \neq 0\}| < \omega \}$, i.e. the functions from β to α with finite support. We may put the following order on $FS(\beta,\alpha)$:

$$f \prec g \quad \text{iff} \quad f(\hat{\gamma}) < g(\hat{\gamma}) \quad \text{where}$$

$\hat{\gamma} = \max\{\gamma \,/\, f(\gamma) \neq g(\gamma)\}$.

Show that, $\forall \alpha, \beta \in \mathrm{Ord}$, $\alpha^\beta \cong \langle FS(\beta,\alpha), \prec \rangle$

(i.e. there exists an order preserving bijection $\psi : \alpha^\beta \to FS(\beta,\alpha)$)

* **2)** Show; Let $\beta, \gamma \leq \aleph_\alpha$. Then $|\alpha + \beta| = |\alpha \cdot \beta| = |\alpha^\beta| \leq \aleph_\alpha$.

3) Show; Let $X \subseteq \aleph_\alpha$ be such that $|X| < \aleph_\alpha$. Then $|\omega_\alpha \setminus X| = \aleph_\alpha$.