Week 3 Thursday

Set Theory

Due Monday Feb 1st

Induction

(prove the following claims)

1) (Taken from Hrbacek and Jech §3.2)

Theorem. (Double Induction)
Let $\phi(x, y)$ be a formula. If $\phi(x, y)$ satisfies:

> (**) $\forall m, n \in \omega$, $\phi(m, n)$ is true if we have $\forall k < m \ \forall \ell \ \phi(k, \ell)$ and $\forall \ell < n \ \phi(m, \ell)$.

Then $\forall m, n \in \omega$, $\phi(m, n)$ is true.

The above form of induction is closely related to the well-ordering of what set?

*__2)__

Consider the following definition of a function $f : \omega \to \omega$:

$$f(n) = \prod_{i \leq n} p_0 \cdot p_1 \cdot \ldots \cdot p_n$$

Where p_i is the ith prime in ω.

Prove the existence of the function by explicitly producing the appropriate sets, constant and function from the statement of the recursion theorem.

*__3)__

Demonstrate the existence (in the same explicit manner as in **2)**) of the fibonacci sequence as given in its usual recursive form:

$$f(0) = 1, \ f(1) = 1, \ \text{and} \ \forall n > 1, \ f(n) = f(n-1) + f(n-2)$$