

Math 75 – Homework

Posted May 23, 2014; due Wednesday, May 28, 2014

1. Suppose a is an integer larger than 1 and p is an odd prime that does not divide $a^2 - 1$. Show that $n = (a^{2p} - 1)/(a^2 - 1)$ is a pseudoprime base a . For example, with $a = 2$ and $p = 5$, we see that $(2^{10} - 1)/(2^2 - 1) = 341$ is a pseudoprime base 2.
2. Prove that a composite number n is a Carmichael number if and only if n is squarefree and for each prime $p \mid n$ we have $p - 1 \mid n - 1$.
3. Using the previous exercise, prove that a Carmichael number must be odd and have at least 3 prime factors.