Math 75 – Homework #9

posted May 22, 2008; due Wednesday, May 28, 2008

Exercises

1. In the lecture notes #22 from May 16 there are several assertions that were not proved. Please prove them. These are

   (a) Lemma 1.

   (b) Lemma 2.

   (c) Lemma 4.

2. Let F be an arbitrary field. Prove that the equation $f^n + g^n = h^n$ has no solutions with $f, g, h \in F[x]$ and $n \in \mathbb{Z}$ provided $n > 2$, f, g, h are coprime, and none of $D(f^n), D(g^n), D(h^n)$ is 0.

3. Prove that the number of polynomials $g(x) \in F_p[x]$ of degree $< 2j$ that are coprime to $h_1(x)h_2(x)$, where h_1, h_2 are different monic irreducible polynomials of degree j is equal to the number of ordered pairs of elements of $\mathbb{F}_{p^j}^{\times}$, namely $(p^j - 1)^2$.

4. Use the algorithm in the notes to find a squareroot of 5 in $\mathbb{F}_{41}$.