Math 75 – Homework #1
posted March 29, 2008; due Monday, March 31, 2008

Exercises

1. Consider the set F^n of all vectors $\mathbf{v}$ with n coordinates and entries in the finite field F of 2 elements. We say vector $\mathbf{v} \in F^n$ is orthogonal to vector $w \in F^n$ if the dot product $v \cdot w$ is 0.

   (a) Show that the codewords in the (8,7) parity check code are exactly the vectors in F^8 orthogonal to $(1, 1, 1, 1, 1, 1, 1, 1)$.

   (b) Find 3 vectors in F^6 such that the codewords for the triple parity check code are exactly those vectors orthogonal to all 3 of your vectors.

   (c) Try to describe the triple repetition code (as described in class) in this way.

2. Show that $\mathbf{Q}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbf{Q}\}$ is a field.

3. Let F be a field.

   (a) Show that if $a, b \in F$ and $ab = 0$, then either $a = 0$ or $b = 0$.

   (b) Suppose A and B are nonzero polynomials over F (that is, nonzero elements of $F[x]$). Suppose A has degree j and B has degree k. Prove that the product AB has degree $k + j$. (We did this in class in the special case when $F = \mathbf{Z}/(3)$.)

4. Let $F = \mathbf{Z}/(2)$, and let $M = x^2 + 1$ and $N = x^2 + x + 1$. Each of the systems $F[x]/(M)$ and $F[x]/(N)$ has four elements. For each system, list the four elements and write out the full 4×4 multiplication table. Exactly one of these two systems is field. Decide which one is not a field and prove that it is not.