Math 74 Spring 2005

Topology II: Introduction to Algebraic Topology

Midterm Exam (take-home)

due Monday May 2 at the end of the lecture

Your name (please print): ____

Instructions: This is an open book, open notes exam. You can use any printed matter (or your class notes) you like but you **can not** consult one another or other humans. **Use of calculators is not permitted**. You must justify all of your answers to receive credit.

The exam total score is the sum of your **5** (out of **6**) best scores. Please do all your work on the paper provided.

The Honor Principle requires that you neither give nor receive any aid on this exam.

Grader's use only

1.	______	/20
2.	______	/20
3.	______	/20
4.	______	/20
5.	______	/20
6.	______	/20
Total:	______	/100

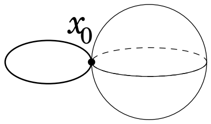
1. Let X be a topological space, $A \subset X$ its **path-connected** subspace and $x_0 \in A$.

a. Define a homomorphism $\pi_1(A, x_0) \to \pi_1(X, x_0)$ induced by the inclusion of A into X;

b. Prove that this homomorphism is surjective **if and only if** every path in X with endpoints in A is homotopic to a path in A.

2. Let $X = S^1 \times I$ be a cylinder and let $s : I \to X$ be a path given by $s(t) = (e^{2\pi i t}, t)$. Is s homotopic (relative boundary!!) to the straight segment connecting points $(1, 0)$ with $(1, 1)$ in X?
Hint: Is X simply-connected? How can this knowledge help?

3. What is the fundamental group of $\mathbb{R}^n \setminus \mathbb{R}^k$ for $n > 1$ and $0 \leq k < n$?
Hint: You may start by figuring out the answer for $k = 0$, 1.

4. Let $X =$ 

be a *bouquet* of S^1 and S^2, that is, a space constructed by identification of exactly one point of S^1 with exactly one point of S^2. Let this new point be x_0.

a. Find (construct explicitly) the universal covering $\widetilde{X}$ of X;

b. Prove that your answer to part a. is indeed a universal covering;

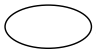
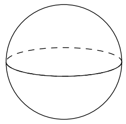
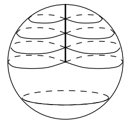
c. Compute the fundamental group $\pi_1(X, x_0)$.
Hint: Have a look at **28.G.3** on page 163. You *might* need to use it at some point.

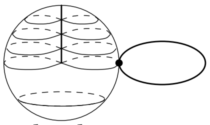
5. Consider the torus $T = S^1 \times S^1$. Let $x_0 = (1, 1) \in T$. We know that $\pi_1(T, x_0) \simeq \mathbb{Z} \times \mathbb{Z}$. Let φ be the corresponding isomorphism.

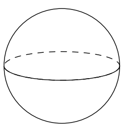
a. Find two loops a and b in T, such that $\varphi([a])$ and $\varphi([b])$ generate $\mathbb{Z} \times \mathbb{Z}$. Depict them on a picture of torus of your choice;

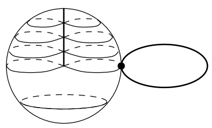
b. Since $\mathbb{Z} \times \mathbb{Z}$ is Abelian, $ab \sim ba$. Find (by presenting an explicit formula) a homotopy between them;

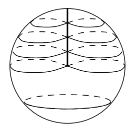
c. Let $k_{n,m} : I \to T$ be a loop given by $k_{n,m}(t) = (e^{2\pi int}, e^{2\pi imt})$. Find $\varphi([k_{n,m}]) \in \mathbb{Z} \times \mathbb{Z}$. Depict the image of $k_{2,3}$ on the torus.

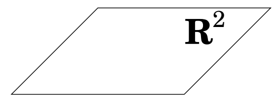
6. In this problem , , and  depict schematically a circle S^1, a sphere S^2 and a real projective plane $\mathbb{R}P^2$, respectively.

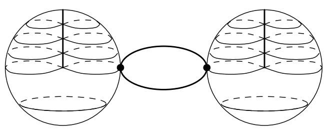
Let $X =$  be a bouquet of $\mathbb{R}P^2$ with S^1. For each of the following spaces figure out whether it can be a covering space for X and **if yes**, than with how many sheets. Don't forget to justify your answers (especially, the negative ones!) and to do the second part of the question.

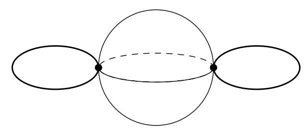
a: 

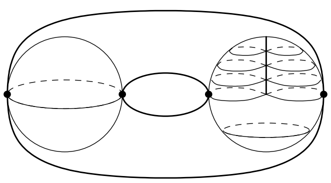
b: 

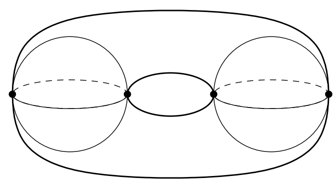
c: 

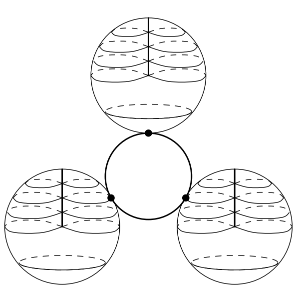
d: 

e: 

f: 

g: 

h: 

i: 

j: