1. Let X be the disjoint union of two copies of the closed interval $[-1, 1]$ in their usual subspace topology. Call them I_1 and I_2. For any $x \in [-1, 1]$ let x_k denotes its representative in I_k. For every $x \neq 0$ let

$$[x] = \{x_1, x_2\}$$

and let $[0]_1 = \{0_1\}$ and $[0]_2 = \{0_2\}$. Let Y be the quotient space formed via these equivalence classes.

   (a) Prove Y is compact.

   (b) Prove Y is locally Euclidean.

   (c) Why is Y not a one dimensional manifold? Prove it

2. Let

$$X = R^2 - \{(0, 0), (1, 0)\}$$

in its usual subspace topology. Now we will decompose X into equivalence classes. When $y \neq 0$ let

$$[x, y] = \{(tx, ty) \mid 0 < t\}$$

and let

$$[-1, 0] = \{(t, 0) \mid t < 0\}$$
$$[1, 0]_s = \{(t, 0) \mid 0 < t < 1\}$$
$$[1, 0]_b = \{(t, 0) \mid 1 < t\}.$$

Let Y be the quotient space formed via these equivalence classes.

   (a) Prove Y is compact.

   (b) Prove Y is locally Euclidean.

   (c) Why is Y not a one dimensional manifold? Prove it.