Handout #2. Differentiable functions. Chain rule

2. Chain rule

Exercise 2.1. If $f : \mathbb{R}^n \to \mathbb{R}^n$ is the identity function, $f(\mathbf{x}) = \mathbf{x}$, what is $Df(\mathbf{x})$?

Exercise 2.2. Consider the functions:

$$f : \mathbb{R}^2 \to \mathbb{R}^2, f(x, y) = (e^x \cos y, e^x \sin y),$$

and

$$g : \{\, (x, y) \,|\, x > 0 \,\} \subset \mathbb{R}^2 \to \mathbb{R}^2, g(x, y) = \left(\frac{1}{2} \ln(x^2 + y^2), \arctan\left(\frac{y}{x}\right) \right).$$

(a) Compute $D(g \circ f)$ at $(0, \pi/4)$ and $(-1, 0)$.
(b) If f bijective? Is g bijective?
(c) What function is $g \circ f$? (At least make a guess ...)

Note. For more about function f see Exercise 7, from §6.