Handout #1. Review of topology of $\mathbb{R}^n$

1. Topology of $\mathbb{R}^n$

Exercise 1.1. Consider the map $]\ [: \mathbb{R}^n \to \mathbb{R}_+$, $]\mathbf{x}[= \sum_1^n |x_i|$.

(a) Show that $]\ [$ is a norm on $\mathbb{R}^n$.
(b) Show that $]\ [$, $\|\ \|$, and $|\ |$ are equivalent norms.
(c) For $n = 2$, make a sketch with the unit balls corresponding of the three norms above.

Exercise 1.2. Show that the topologies generated on a vector space V by two equivalent norms are the same.

Exercise 1.3. We defined a set U of a metric space to be *open* if

for every point $x_0 \in U$ there exists $\varepsilon > 0$ such that $U(x_0; \varepsilon) \subset U$.

Show that the open sets satisfy the axioms (τ1), (τ2), and (τ3) of a topology.

Exercise 1.4. Let f be a function between metric spaces. Give the ε-δ definition of

$$\lim_{x \to x_0} f(x) = y_0.$$

Exercise 1.5.

(a) Let A be a subset of the topological space X. Show that $x \in \mathrm{Bd}A$ iff every open set containing x intersects both A and $X \setminus A$.
(b) Find the boundary of the unit ball $U(\mathbf{0}, 1)$ and half-space $\mathbb{H} = \{\, \mathbf{x} \in \mathbb{R}^n \mid x_n \geq 0 \,\}$.