# Math 71

Homework Assignment 25 - 29, October 1999

1. p. 124: 8, 10, 14
2. Let $G$ be a group of order 105. Show that $G$ has both a normal Sylow 5- subgroup and a normal Sylow 7-subgroup.
3. Let $G$ be a group of order 48. Show that $G$ has a normal subgroup of order 8 or 16.
4. Let $G$ be a group of order 231, and suppose that $G$ has only one Sylow 3-subgroup. Show that $G$ is cyclic.

**A few hints...**

For problem 2:

(a) First show that if $H$ is a group of order 35, all its Sylow $p$-subgroup s are normal in $H$ (i.e. $n_5 = n_7 = 1$).

(b) Next show that if $G$ is a group of order 105, for at least one of $p = 5$ or $p = 7$, we have $n_p = 1$.

(c) For each $p = 5, 7$, let $H_p$ denote a fixed Sylow $p$-subgroup of $G$. Show that $H = H_5 H_7$ is a normal subgroup of $G$.

(d) Let $P$ be any Sylow $p$-subgroup of $G$, $p = 5$ or 7. Show that $P = H_p$.

For problem 3:

(a) If there is more than one Sylow 2-subgroup, let $H$ and $K$ be any two of them. Show that $|H \cap K| = 8$.

(b) Show that $H, K \subset N_G(H \cap K)$.

(c) Show that $G = N_G(H \cap K)$.