Your Name Here

Math 71

Homework 6

1. Show that every nonabelian group of order 6 has a nonnormal subgroup of order 2. Use this to classify all groups of order 6. **Note:** This is the exercise you prove to justify the statement we have been using all term, that there are exactly two isomorphism classes of groups of order 6. So do not invoke that result here.

2. Show that for $n \geq 3$ and p an odd prime, that every Sylow p-subgroup of D_{2n} is cyclic and normal in D_{2n}.

3. List all the Sylow 2-subgroups of S_4, and given two of them, find an element σ is S_4 which conjugates one to the other. **Hint**: Can you find a permutation τ so that $\tau(a_1\, a_2\, \ldots\, a_r)\tau^{-1} = (b_1\, b_2\, \ldots\, b_r)$? Parts of §4.3 that we did not cover in class have some interesting results!

4. Let G be a group of order $105 = 3 \cdot 5 \cdot 7$. For each prime p dividing 105, let H_p denote a Sylow p-subgroup of G.

   - Show that $n_5 = 1$ or $n_7 = 1$.
   - Use that to conclude that $H_5 H_7$ is a normal subgroup of G and that $n_5 = n_7 = 1$.
   - Show that if $n_3 = 1$, then G is cyclic.

5. Determine all the distinct isomorphism classes of abelian groups of order $9801 = 3^4 \cdot 11^2$. Give two lists, one using elementary divisors and the other using invariant factors.