Math 71

Homework 6

1. page 122: 10, 14
2. page 165: #2,3(b,e)
3. Let G be a group of order 48. Show that G has a normal subgroup of order 8 or 16. Some hints are provided.
   (a) If there is more than one Sylow 2-subgroup, let H and K be any two (distinct) Sylow 2-subgroups. Show that $|H \cap K| = 8$.
   (b) Show that for the H and K above, that $H, K \subseteq N_G(H \cap K)$.
   (c) Show that $G = N_G(H \cap K)$.
4. Let G be a group of order $231 = 3 \cdot 7 \cdot 11$, and suppose that there is only one Sylow 3-subgroup. Show that G is cyclic.