Final Practice Problems for Math 71

1. Let F be a field and $a \in F \setminus \{0\}$.

   (a) If $ap(x)$ is irreducible in $F[x]$, then prove that $p(x)$ is irreducible over F.

   (b) If $p(ax)$ is irreducible in $F[x]$, then prove that $p(x)$ is irreducible over F.

   (c) If $q(x) := p(x + a)$ is irreducible in $F[x]$, then prove that $p(x)$ is irreducible over F.

   (d) Use part (c) to show that $p(x) = 8x^3 - 6x + 1$ is irreducible over $\mathbf{Q}$

2. Which of the following polynomials are irreducible over $\mathbf{Q}$?

   (a) $x^5 + 9x^4 + 12x^2 + 6$.

   (b) $x^4 + x + 1$.

   (c) $x^4 + 3x^2 + 3$.

   (d) $x^5 + 5x^2 + 1$.

   (e) $x^4 + 1$.

   (f) $(\frac{5}{2})x^5 + (\frac{9}{2})x^4 + 15x^3 + (\frac{3}{7})x^2 + 6x + \frac{3}{14}$.

3. We showed in lecture that $p_n(x) := x^{n-1} + x^{n-1} + \cdots + x + 1$ is irreducible over $\mathbf{Q}$ if n is prime. Show that $p_n(x)$ is irreducible if and only if n is prime.