Math 71 Homework.
Do Not Turn In

1. Suppose that $f(x) \in \mathbf{Z}_p[x]$ is irreducible and that $\deg f(x) = n$. Prove that $\mathbf{Z}_p[x]/(f(x))$ is a field with p^n elements.

2. Construct a field with 25 elements.

3. Write $x^3 + 6$ in $\mathbf{Z}_7[x]$ as a product of irreducibles in $\mathbf{Z}_7[x]$.

4. If r is a real number such that $r + \frac{1}{r}$ is an odd integer, then show that r must be irrational.