

1. Suppose that D is an integral domain and that $F = \{a/b : a, b \in D \text{ and } b \neq 0\}$ is the field of fractions in D constructed in lecture. Let $\iota : D \rightarrow F$ be given by $\iota(a) := a/1$. Show that if $\phi : D \rightarrow Q$ is an *injective* ring homomorphism into a field Q , then there is an injective ring homomorphism $\bar{\phi} : F \rightarrow Q$ such that

$$\begin{array}{ccc}
 D & \xrightarrow{\phi} & Q \\
 & \searrow \iota & \nearrow \bar{\phi} \\
 & & F
 \end{array}$$

commutes. *I forgot to insist that ϕ should be injective in the version of the theorem I stated in lecture.* Show that if ϕ is not injective, then there is no homomorphism $\bar{\phi}$ that makes the diagram commute.