## Math 71

Homework 6

- page 122: 8, 10, 14
- pp 165-166: 2(b,e), 3(b,e)
- The following additional problems:

1. Let $G$ be a group of order 48. Show that $G$ has a normal subgroup of order 8 or 16. Some hints are provided.
(a) If there is more than one Sylow 2-subgroup, let $H$ and $K$ be any two (distinct) Sylow 2-subgroups. Show that $|H \cap K|=8$.
(b) Show that for the $H$ and $K$ above, that $H, K \subseteq N_{G}(H \cap K)$.
(c) Show that $G=N_{G}(H \cap K)$.
2. Let $G$ be a group of order $231=3 \cdot 7 \cdot 11$, and suppose that there is only one Sylow 3-subgroup. Show that $G$ is cyclic.
