Due Wednesday Nov. 9th in class.

You may use your class notes, the on-line solutions, and the textbook. All work should be your own. No other persons, books, or web pages should be consulted.

Do any six of the eight problems. You may do one or two of the remaining problems for extra credit. If you do work more than six problems, please indicate which are for extra credit. (Partial credit will be harder to get on the extra credit problems.)

1.
   (a) Show that if $\varphi: G \to G'$ is a homomorphism of groups and $H' \leq G'$, then $\varphi^{-1}(H') \leq G$.
   (b) Now assume that $K$ is a normal subgroup of $G$, and let $\pi: G \to G/K$ be the usual homomorphism. If $K \leq H \leq G$ and $H' = \pi(H) = H/K$ is the subgroup of $G/K$ corresponding to $H$, show that $H$ is normal in $G$ if and only if $H'$ is normal in $G/K$
2. Show that the dihedral group $D_6$ is isomorphic to the direct product of two nontrivial groups.
3. page 73, number 19: Prove that if a group contains exactly one element of order 2, then that element is in the center of the group.
4. Let $G$ b a nonabelian group of order $p^3$, where $p$ be a prime.
   (a) Prove that $Z(G)$, the center of $G$ has order $p$, and that $G/Z(G) \approx \mathbf{Z}_p \times \mathbf{Z}_p$.
   (b) If $H$ is a subgroup of $G$ of order $p^2$, show that $Z(G) \subseteq H$ and that $H$ is normal in $G$.
   (c) Assume $x^p = 1$ for all $x \in G$. Show $G$ contains a normal subgroup isomorphic to $\mathbf{Z}_p \times \mathbf{Z}_p$.
5. Classify all abelian groups of order $p^5q^2$ where $p$ and $q$ are distinct primes.
6. Let $G$ be a group of order 48. Show that $G$ has a normal subgroup of order either 8 or 16.
7. Let $C$ be the group of symmetries (or rigid motions) of a cube, as discussed in class.
   (a) Give an explicit description of $C$ as a subgroup of $SL_3(\mathbf{R})$.
   (b) Show that $C$ is isomorphic to one of the groups (other than $C$ itself) we've considered. Hint: Consider the action of $C$ on the set of long diagonals of a cube.
   (c) Find an index 2 subgroup of $H$. Specifically, viewing $C$ as a subgroup of $SL_3(\mathbf{R})$, say which matrices make up the subgroup $H$.
8. Let $H = \{\pm 1, \pm i, \pm j, \pm k\}$ be the quaternion group, and let $Aut(H)$ be its group of automorphisms. Show that $Aut(H)$ is isomorphic to one of the groups we've considered.