1. Indicate which of the following sentences are statements.

    (a) 1024 is the smallest four-digit number that is a perfect square.

    (b) She is a mathematics major.

    (c) $128 = 2^6$

    (d) $x = 2^6$

    (e) This sentence is false.

2. Let p be the statement "DATAENDFLAG is off," q the statement "ERROR equals 0," and r the statement "SUM is less than 1000." Express the following sentences in symbolic notation.

    (a) DATAENDFLAG is off, ERROR equals 0, and SUM is less than 1000.

    (b) DATAENDFLAG is off but ERROR is not equal to 0.

    (c) DATAENDFLAG is off; however ERROR is not 0 or SUM is greater than or equal to 1000.

    (d) DATENDFLAG is on and ERROR equals 0 but SUM is greater than or equal to 1000.

    (e) Either DATENDFLAG is on or it is the case that both ERROR equals 0 and SUM is less than 1000.

3. Write the negation for each of the following statements.

    (a) Hal is a math major and Hal's sister is a computer science major.

    (b) The connector is loose or the machine is unplugged.

    (c) If today is Thanksgiving, then tomorrow is Friday.

    (d) If n is prime, then n is odd or n is 2.

    (e) If Tom is Ann's father, then Jim is her uncle and Sue is her aunt.

    (f) ∀ computer programs P, if P is correct then P compiles without error messages.

    (g) ∀ integers a, b, and c, if $a - b$ is even and $b - c$ is even, then $a - c$ is even.

    (h) ∀ animals x, if x is a cat then x has whiskers and x has claws.

4. Rewrite the following statements in if-then form.

    (a) This loop will repeat exactly N times if it does not contain a "stop" or a "go to."

    (b) Catching the 8:05 bus is a sufficient condition for my being on time for work.

    (c) A grade-point average of at least 3.7 is a necessary condition for graduating with honors.

5. True or false? The negation of "If Sue is Luiz's mother, then Deanna is his cousin" is "If Sue is Luiz's mother, then Deanna is not his cousin."

6. Write the contrapositive of each of the following statements.

    (a) If today is Thanksgiving, then tomorrow is Friday.

    (b) If n is prime, then n is odd or n is 2.

    (c) If Tom is Ann's father, then Jim is her uncle and Sue is her aunt.

    (d) ∀ computer programs P, if P is correct, then P compiles without error messages.

    (e) ∀ animals A, if A is a cat then A has whiskers and A has claws.

7. Fill in the blanks using modus ponens or modus tollens to produce valid arguments.

    (a) If $\sqrt{2}$ is rational, then $\sqrt{2} = a/b$ for some integers a and b.
    It is not true that $\sqrt{2} = a/b$ for some integers a and b.
    ∴ _____

(b) If this is a "while" loop, then the body of the loop may never be executed.

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∴ The body of the loop may never be executed.

(c) If logic is easy, then I am a monkey's uncle.

I am not a monkey's uncle.

∴ ———————————————

(d) If this polygon is a triangle, then the sum of its interior angles is 180°.

The sum of the interior angles of this polygon is not 180°.

∴ ———————————————

(e) If they were unsure of the address, then they would have telephoned.

———————————————————

∴ They were sure of the address.

8. Determine if the following argument is valid. If it is not valid, explain why not.

If Jules solved this problem correctly, then Jules obtained the answer 2.

Jules obtained the answer 2.

∴ Jules solved this problem correctly.

9. In the back of an old cupboard you discover a note signed by a pirate famous for his bizarre sense of humor and love of logic puzzles. In the note he wrote that he had hidden treasure somewhere on the property. He listed five true statements (given below) and challenged the reader to use them to figure out the location of the treasure.

(a) If this house is next to a lake, then the treasure is not in the kitchen.

(b) If the tree in the front yard is an elm, then the treasure is in the kitchen.

(c) This house is next to a lake.

(d) The tree in the front yard is an elm or the treasure is buried under the flagpole.

(e) If the tree in the backyard is an oak, then the treasure is in the garage.

Where is the treasure hidden?

10. A menagerie consists of seven brown dogs, two black dogs, six gray cats, ten black cats, five blue birds, six yellow birds, and one black bird. Determine which of the following statements are true and which are false.

(a) There is an animal in the menagerie that is red.

(b) Every animal in the menagerie is a bird or a mammal.

(c) Every animal in the menagerie is brown or gray or black.

(d) There is an animal in the menagerie that is neither a cat nor a dog.

(e) No animal in the menagerie is blue.

(f) There are a dog, a cat, and a bird in the menagerie that all have the same color.

11. Consider the following statement:

∀ basketball players x, x is tall.

Which of the following are equivalent ways of expressing this statement?

(a) Every basketball player is tall.

(b) Among all the basketball players, some are tall.

(c) Some of all the tall people are basketball players.

(d) Anyone who is tall is a basketball player.

(e) All people who are basketball players are tall.

(f) Anyone who is a basketball player is a tall person.

12. Rewrite each of the following statements in the form "∀_____x, _____."

(a) All dinosaurs are extinct.

(b) Every real number is positive, negative, or zero.

(c) No irrational numbers are integers.

(d) No logicians are lazy.

13. Rewrite each of the following in the form "$\exists$_____ x such that _____."

   (a) Some exercises have answers.

   (b) Some real numbers are rational.

14. Which of the following is a negation for "Every polynomial function is continuous"? More than one answer may be correct.

   (a) No polynomial function is continuous.

   (b) Some polynomial functions are not continuous.

   (c) Every polynomial function fails to be continuous.

   (d) There is a noncontinuous polynomial function.

15. In each of the following, determine whether the proposed negation is correct. If not, write a correct negation.

   (a) *statement*: The sum of any two irrational numbers is irrational.

     proposed negation: The sum of any two irrational numbers is rational.

   (b) *statement*: For all integers n, if n^2 is even then n is even.

     proposed negation: For all integers n, if n^2 is even then n is not even.

16. For each of the following, (i) rewrite the statement in English without using the symbols $\forall$ or $\exists$, and (ii) write a negation for the statement in symbols and in English.

   (a) $\forall$ colors C, $\exists$ an animal A such that A is colored C.

   (b) $\exists$ a book b such that $\forall$ people p, p has read b.

17. Consider the statement "Everybody is older than somebody." Rewrite this statement in the form "$\forall$ people x, $\exists$_____."

18. Consider the statement "Somebody is older than everybody." Rewrite this statement in the form "$\exists$ a person x such that _____."

19. For each of the following statements, (i) rewrite the statement formally using quantifiers and variables, and (ii) write a negation for the statement in symbols and in English.

   (a) Everybody trusts somebody.

   (b) Somebody trusts everybody.

   (c) Any even integer equals twice some other integer.

   (d) There is a program that gives the correct answer to every question that is posed to it.

20. A group of 100 workers were asked if they were college graduates and if they belonged to a union. Sixty were not college graduates, 20 were nonunion college graduates, and 30 were union members. How many of the workers were neither college graduates nor union members?

21. One of Shakespeare's sonnets has a verb in 11 of its 14 lines, an adjective in 9 lines, and both in 7 lines. How many lines have a verb but no adjective? An adjective and no verb? Neither an adjective nor a verb?

22. A chemical engineer wishes to observe the effects of temperature, pressure, and catalyst concentration on the yield resulting from a certain reaction. If she intends to include two different temperatures, three pressures, and two levels of catalyst, how many different runs must she make in order to observe each temperature-pressure-catalyst combination exactly twice?

23. A combination lock requires three selections of numbers, each from 1 through 20.

   (a) How many different combinations are possible?

   (b) Suppose the locks are constructed in such a way that no number can be used twice. How many different combinations are possible?

24. A coded message from a CIA operative to his Russian KGB counterpart is to be sent

in the form Q4ET, where the first and last entries must be consonants; the second, an integer 1 through 9; and the third, one of the six vowels. How many different ciphers can be transmitted?

25. An octave contains 12 distinct notes (on a piano, five black keys and seven white keys). How many different eight-note melodies within a single octave can be written using the white keys only? How many different eight-note melodies within a single octave can be written if the black keys and white keys need to alternate?

26. How many arrangements of no more than three letters can be formed using the letters of the word NETWORK (with no repetitions allowed)?

27. An early BASIC compiler recognized variable names according to the following rules: Numeric variable names had to begin with a letter and then the letter could be followed by another letter or a digit or by nothing at all. String variable names had to begin with the symbol $ followed by a letter, which could then be followed by another letter or a digit or by nothing at all. How many distinct variable names were recognized by this BASIC compiler?

28. The nine members of the music faculty baseball team, the Mahler Maulers, are all incompetent and each can play any position equally poorly. In how many different ways can the Maulers take the field?

29. A three-digit number is to be formed from the digits 1 through 7, with no digit being used more than once. How many such numbers would be less than 289?

30. An engineer needs to take three technical electives sometime during his final four semesters. The three are to be selected from a list of 10. In how many ways can he schedule those classes, assuming that he never wants to take more than one technical elective in any given term?

31. A student council consists of 15 students.

(a) In how many ways can a committee of six be selected from the membership of the council?

(b) Two council members have the same major and are not permitted to serve together on a committee. How many ways can a committee of six be selected from the membership of the council?

(c) Two council members always insist on serving on committees together. If they can't serve together, they won't serve at all. How many ways can a committee of six be selected from the council membership?

(d) Suppose the council contains eight men and seven women.

   i. How many committees of six contain three men and three women?

   ii. How many committees of six contain at least one woman?

(e) Suppose the council consists of three freshmen, four sophomores, three juniors, and five seniors. How many committees of eight contain two representatives from each class?

32. An instructor gives an exam with fourteen questions. Students are allowed to choose any ten to answer.

(a) How many different choices of ten questions are there?

(b) Suppose six questions require proof and eight do not.

   i. How many groups of ten questions contain four that require proof and six that do not?

   ii. How many groups of ten questions contain at least one that requires proof?

   iii. How many groups of ten questions contain at most three that require proof?

(c) Suppose the exam instructions specify that at most one of questions 1 and 2 may be included among the

ten. How many different choices of ten questions are there?

(d) Suppose the exam instructions specify that either both questions 1 and 2 are to be included among the ten or neither is to be included. How many different choices of ten questions are there?

33. An all-male club is considering opening its membership to women. In a preliminary survey on the issue, 19 of the 30 members favored admitting women and 11 did not. A committee of six is to be chosen to give further study to the issue.

(a) How many committees of six can be formed from the club membership?

(b) How many of the committees will contain at least three men who, in the preliminary survey, favored opening the membership to women?

34. Suppose that three computer boards in a production run of forty are defective. A sample of four is to be selected to be checked for defects.

(a) How many different samples can be chosen?

(b) How many samples will contain at least one defective board?

35. On an 8×8 chessboard, a rook is allowed to move any number of squares either horizontally or vertically. How many different paths can a rook follow from the bottom-left square of the board to the top-right square of the board if all moves are to the right or upward?

36. The crew of *Apollo 17* consisted of two pilots and one geologist. Suppose that NASA had actually trained a total of nine pilots and four geologists. How many possible *Apollo 17* crews could have been formed?

(a) Assume that the two pilot positions have identical duties.

(b) Assume that the two pilot positions are really a pilot and a copilot.

37. The Alpha Beta Zeta sorority is trying to fill a pledge class of nine new members during fall rush. Among the 25 available candidates, 15 have been judged marginally acceptable and 10 highly desirable. How many ways can the pledge class be chosen to give a two-to-one ratio of highly desirable to marginally acceptable candidates?

38. The final exam in History 101 consists of five essay questions that the professor chooses from a pool of seven that are given to the students a week in advance. For how many possible sets of questions does a student need to be prepared? In this situation does order matter?

39. Nine students, five men and four women, interview for four summer internships sponsored by a city newspaper.

(a) In how many ways can the newspaper choose a set of four interns?

(b) In how many ways can the newspaper choose a set of four interns if it must include two men and two women in each set?

(c) How many sets of four can be picked such that not everyone in a set is of the same sex?

40. Ten basketball players meet in the school gym for a pickup game. How many ways can they form two teams of five each?

41. A chemist is trying to synthesize part of a straight-chain aliphatic hydrocarbon polymer that consists of 21 radicals—10 ethyls (E), 6 methyls (M), and 5 propyls (P). Assuming all arrangements of radicals are physically possible, how many different polymers can be formed if no two of the methyl radicals are to be adjacent?

42. A boat has a crew of eight: Two of those eight can row only on the stroke side, while three can row only on the bow side. In how many ways can the two sides of the boat be manned if there are four people on each side?