April 20 – Lecture 10

Simple and Compound Interest.

Example 1:

Suppose you want to buy a car, but have no cash on hand. You need the car to get to your new job, and you know you will be able to save $8000 by the end of the year (8 months from now). Your mom wants to help you out, but wants you to know that you are an adult so has asked that you pay her back, with 1% nominal interest, on January 1.

If you borrow $8000, how much will you have to pay back?

This is a question of simple interest, which is what is used for loans with a duration less than a year or for family loans (problems in this course will specify if simple interest will be used). For simple interest, the ~~rate of~~ interest is charged only for the principal amount, and is a yearly rate.

The interest charged by your mother is $I = Prt$ where P is the principal

(\$8000), r is the interest rate (.01) and t is the time, in years ($8/12 = 2/3$). Your interest is:

$$I = (8000)(.01)(2/3) = \$53.33$$

So you would need to pay \$8053.33 on January 1.

Your mom may not know that she should only charge you interest for the ~~percent~~ portion of the year for which the money was borrowed, but since this is to show you ~~are~~ know you are an adult, if you explain simple interest to her she'll probably go with it.

For simple interest, the amount due at the end of the loan period is

$$A = P(1 + rt)$$

When calculating the time, it is common to assume 360 days a year (12 months of 30 days each).

~~For~~ What if you know you will have \$8000, but don't know that you will have the interest. How much should you borrow?

Now, the \$8000 should be the amount

you pay back, A.

So $8000 = P(1 + (.01)(\frac{2}{8}))$

$P = \$7947.02$

You can borrow \$7947 and still be sure you can pay her back.

The more frequent type of interest is <u>compound interest</u>. For compound interest, interest is charged on both the principal and ~~past~~ interest that has been charged in the past.

We will begin with the assumption that this is your trust fund: no money (except the interest) is being deposited or withdrawn.

Example 2: 18 years ago, your grandmother deposited \$5000 as a college fund, with 7% interest compounded annually. How much money is there now?

after 1 year: $P_1 = P_0(1+r)$

$P_1 = 5000(1+.07)$

$= 5350$

after 2 years: $P_2 = P_1(1+r) = P_0(1+r)(1+r)$

$= 5724.50$.

after 18 years: $P_{18} = P_{17}(1+r) = P_0(1+r)^{18}$

$= \$16,899.66$

(almost enough for 1 term!)

In general, for interest which is compounded annually, the amount A after t years with annual interest r is given by

$$A = P(r+1)^t.$$

For interest which is compounded m times a year, the formula is

$$A = P\left(\cancel{t} + \frac{r}{m}\right)^{mt}.$$

This is slightly different from the formula in the book: they use a periodic rate, $i = \frac{r}{m}$, for the interest and look at the number of times the interest is compounded, $n = mt$. I think this formula is easier because there are fewer things to remember.

If your grandmother had deposited the same \$5000 in an account which compounded monthly, after 18 years you would have

$$A = \$5000\left(1 + \frac{.07}{12}\right)^{18 \cdot 12}$$
$$= \$17,562.70$$

(still ~~not~~ enough for a year).

When interest is compounded more than annually, there is an annual rate (r) and an effective rate (r_e).

r_e, the effective rate, corresponds to the amount of interest you would get if the account were compounded annually. In other words,

$$P\left(1+\frac{r}{m}\right)^{mt} = P(1+r_e)^t$$

We can get a formula for r_e:

$$r_e = \left(1+\frac{r}{m}\right)^m - 1$$

This can make comparing accounts easier.

Example 3: For a car loan, you can borrow from the bank (5.2%, compounded quarterly) or from the dealer (4.9%, compounded monthly). Which should you choose?

If $r = 5.2\%$, $m = 4$,

$$r_e = \left(1+\frac{.052}{4}\right)^4 - 1 = .053 = 5.3\%$$

If $r = 4.9\%$, $m = 12$

$$r_e = \left(1+\frac{.049}{12}\right)^{12} = .050 = 5.0\%$$

The dealer has a better deal.

The book goes through several examples where they solve for the ~~term~~ investment amount when they know the final amount. Here's one:

Example 4:
You want to invest in a trust fund for your newborn daughter.
Suppose college tuition goes up 12% a year and 1 year at a private school currently costs \$30,000. You found an account which will give ~~8~~ 10% interest, compounded quarterly. How much should you invest?

The cost of college will be:

$$30{,}000(1+.12)^{18} + 30{,}000(1+.12)^{19} + 30000(1+.12)^{20} + 30000(1+.12)^{21}$$

$$\$1{,}102{,}600$$

So we want \$1.2 million in the account in 18 years. $A = 1{,}200{,}000$, $t = 18$, $m = 4$, $r = .10$

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$P = \frac{A}{\left(1+\frac{r}{m}\right)^{mt}} = \$202{,}800$$