

**Math 6**  
**Worksheet 7**  
**Due Monday May 17**

Please discuss the following questions with your assigned groups. You may take notes on all items you discuss with your classmates, however you are to write up your solutions independently of one another and without assistance. Your solutions should be written up carefully and neatly on a separate sheet of paper. You should write in **complete sentences** and **explain all steps taken and tools used (such as theorems or results from class) in reaching your final answers**. Please also include at the top of your write-up a list of people with whom you discussed these problems.

The games on this worksheet are examples of *combinatorial games*, which are a particular type of two player game. They are analyzed without the use of payoff matrices. These are each different versions of a game called *Nim*.

1. *Chips Take-Away Game*. Here are the rules of a simple game of removing chips from a pile of chips:
  - a. There are two players. We label them I and II.
  - b. There is a pile of 14 chips in the center of a table.
  - c. A move consists of removing one, two, or three chips from the pile. At least one chip must be removed, but no more than three may be removed.
  - d. Players alternate moves with Player I starting.
  - e. The player that removes the last chips wins. (That is, the last player to move wins. If you can't move, you lose.)

Analyze this game by answering the following questions: Can one player force a win in this game? Which player would you rather be - the player who starts or the player who goes second? What is a good strategy?

*You may find it helpful to start by analyzing the game starting with a smaller number of chips in the pile.*

2. *"Two-Pile" Chips Take-Away Game*. The rules to this game are the same as the rules above, except rules b and c are as follows:
  - b. There are two piles of 7 chips each.
  - c. A move consists of removing one, two, or three chips from one pile. You may remove chips from either pile on any move, but not some chips from both piles on a single move. At least one chip must be removed, but no more than three may be removed.

Analyze this game by answering the following questions: Can one player force a win in this game? Which player would you rather be – the player who starts or the player who goes second? What is a good strategy?

**\*\*\*Bonus\*\*\*** *“Three-Pile” Chips Take-Away Game*: The rules to this game are the same as the rules above, except rules b and c are as follows:

- d. There are three piles of 4 chips each.
- e. A move consists of removing one, two, or three chips from one pile. You may remove chips from any pile on any move, but not some chips from each on a single move. At least one chip must be removed, but no more than three may be removed.

Analyze this game by answering the following questions: Can one player force a win in this game? Which player would you rather be – the play who starts or the player who goes second? What is a good strategy?