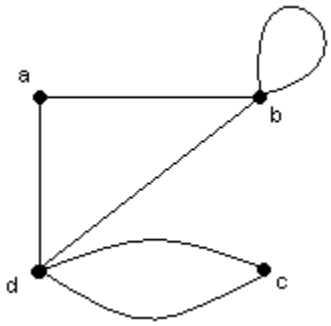
Math 6
Worksheet 6
Due Monday May 10

Please discuss the following questions with your assigned groups. You may take notes on all items you discuss with your classmates, however you are to write up your solutions independently of one another and without assistance. Your solutions should be written up carefully and neatly on a separate sheet of paper. You should write in **complete sentences** and explain all steps taken and tools used (such as theorems or results from class) in reaching your final answers. *Please also include at the top of your write-up a list of people with whom you discussed these problems.*

1. An *odd cycle* is a cycle that has an odd number of edges in it. Explain why every bipartite graph will never have any odd cycles in it. You may wish to look at some examples but be sure that you explain why this is true for *all* bipartite graphs and not just for a few examples.

2. The *adjacency matrix*, A, of a graph G is the matrix obtained by letting the entry in the i^{th} row and the j^{th} column be the number of edges between the i^{th} and j^{th} vertices. For example:

G =



$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

Note: *For the following questions, as in problem 1, it may be helpful to look at some simple examples, but I want you to explain why your answers are true in general.*

   a. For any graph G, its adjacency matrix A is a square matrix (i.e., it has the same number of rows and columns), so we know that we can form the product $A \cdot A$. We also use the notation A^2 to denote $A \cdot A$. For any graph G, what do the entries of the matrix A^2 tell us about the graph?

   b. More generally, if A^m denotes the matrix product $A \cdot A \cdot A \cdot \ldots \cdot A$ (with m terms in the product), what do the entries of the matrix A^m tell us about the graph G?

c. What do the entries of the matrix $A+A^2+A^3+...+A^m$ tell us about the graph G?

d. Assume that for some graph, H, and its adjacency matrix, A, we know that A^6 has only positive integer entries (that is, none of the entries is zero). What can we conclude about H? Explain what led you to this conclusion.