Math 6
Worksheet 5
Due Monday May 3

Please discuss the following questions with your assigned groups. You may take notes on all items you discuss with your classmates, however you are to write up your solutions independently of one another and without assistance. Your solutions should be written up carefully and neatly on a separate sheet of paper. You should write in **complete sentences** and explain all steps taken and tools used (such as theorems or results from class) in reaching your final answers. *Please also include at the top of your write-up a list of people with whom you discussed these problems.*

<u>**Note:**</u> *All graphs in this worksheet are* **simple** *graphs. That is, all graphs have no parallel edges and no loops.*

1. A graph is called <u>*complete*</u> if there is an edge between every pair of vertices. How many edges are there in a complete graph with *n* vertices?

2. You are holding a dinner party and want to ensure that there will be either at least three mutual acquaintances or at least three mutual strangers. Show that this will be the case if you invite six people.

    -   It may be helpful to think about this in terms of a complete graph with edges colored with two different colors
    -   You may also want to begin by singling out one person and looking at what his relationships could be.

    Bonus
    Show that you must invite at least six people for the condition to hold. That is, if you invite fewer people, show that it may not be the case that there are three mutual acquaintances or strangers.

3. The *Pigeonhole Principle* states the following:

    > *If* more than n *pigeons perch in* n *pigeonholes, then at least one pigeonhole will have more than one pigeon in it.*

    Use the Pigeonhole Principle to show that every graph (with at least two vertices) will have at least two vertices with the same degree.