Math 6
Worksheet 2
Due Monday April 12

Please discuss the following questions with your assigned groups. You may take notes on all items you discuss with your classmates, however you are to write up your solutions independently of one another and without assistance. Your solutions should be written up carefully and neatly on a separate sheet of paper. You should write in **complete sentences** and explain all steps taken and tools used (such as theorems or results from class) in reaching your final answers. Please also include at the top of your write-up a list of people with whom you discussed these problems.

1) In class last week, we used Venn Diagrams to come up with an expression for $n(A \cup B)$ where A and B are sets. We concluded that

$$n(A \cup B) = n(A) + n(B) - n(A \cap B),$$

and we called this the Inclusion-Exclusion Principle (for two sets). Can you come up with an Inclusion-Exclusion Principle for three sets? In other words, given three sets A, B and C, use Venn Diagrams to come up with a similar type of expression for $n(A \cup B \cup C)$.

2) How many ways are there for a horse race with four horses to finish if ties are possible? (Note that since ties are possible, any number of the four horses may tie.)

3) Pascal's Identity says that

$$\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}.$$

Recall that $\binom{n}{r}$ is the number of r element subsets of an n element set. Prove Pascal's Identity by counting the number of r element subsets of an n element set in a *different* way to come up with $\binom{n-1}{r} + \binom{n-1}{r-1}$.

Hints:
(1) You may find it helpful to look at some examples with small values of n and r.
(2) Notice the "+ "on the right hand side of Pascal's Identity. You'll probably want to incorporate a statement that involves the word "or."
(3) At some point, it may be helpful to single out one of the elements of your n element set.

(over)

*****Challenge Problem (Bonus points)*****

Use an argument similar to your answer to number 3 to show that

$$P(n, r) = P(n\text{-}1, r) + r \cdot P(n\text{-}1, r\text{-}1).$$