Syllabus - Math 68

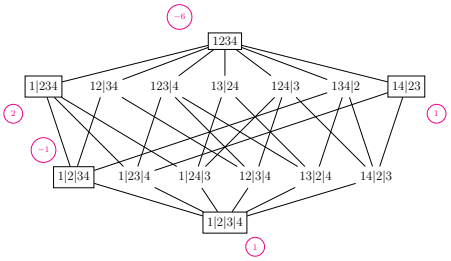
Fall 2019

Lecturer: Nadia Lafrenière (Kemeny Hall 318, nadia.lafreniere@dartmouth.edu)
Class schedule: MWF 11:30 - 12:35 PM, Room TBD, 100C for the first week
x-hour: Tu 12:15-1:05 PM, Room TBD
Website for the class: `https://math.dartmouth.edu/~m68f19`
Office hours: Monday 10-11, WF 9-11, Room 229
Upon approval by the students

Course Objectives

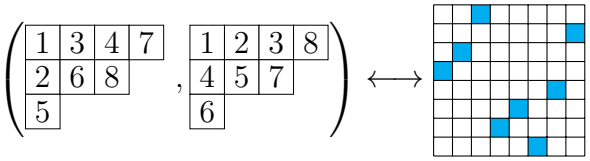
Algebraic combinatorics is defined as the interactions between algebra and combinatorics. Techniques from algebra may solve combinatorial problem and conversely.
The goal of this class is to introduce some notions of combinatorics and to use the techniques from it along with linear and abstract algebra. The first part of the course will be dedicated to review principles from enumerative combinatorics, such as basic statistics, counting principles and generating functions. Going forward, we will apply these to three main topics:

- Partial ordered sets (posets), lattices, Möbius function and incidence algebra. Sperner's lemma.



- Group acting on a set and Pólya theory.

- Representation theory of the symmetric group and Young tableaux. Robinson-Schensted bijection and applications to permutation patterns. Promotion and evacuation, branching rule.



Teaching Methods & Philosophy

Throughout the semester, the blackboard will be the main tool for in-class teaching. That being said, I would like to make some computer demonstrations and I plan to include some time for the students to work on short problems together. Some non-graded and anonymous quizzes will be held during the class, and this is for me and you to know what you understood well and what should be covered more deeply.

Attendance Policy and x-hour

Attendance to class is highly recommended, but not evaluated. However, if you are missing a class, you are responsible for catching up on what was taught. If you know that you will not be able to deliver an evaluation on time *for a serious reason* (illness, sports league tournament, dependents needing care), let me know as soon as possible. The x-hour will be useful for complements to the course. By this, I essentially mean background material if it is needed only for some students, and some introduction on how to use software for doing mathematics. It is not mandatory that you attend it, but it is recommended if you are unfamiliar with what will be taught during this time. The x-hour will not be used every week, and I will advertise in advance the content of it.

Content's Schedule (tentative)

This schedule is tentative, and the most up-to-date version will be posted on the course webpage. The list of references for each part might change as well, but you will find most of the content taught in the references that are listed here.
Homework assignments are due on Wednesdays, except for the first week.

Date	**Content**	**References**
September 16	Introduction	
September 18	What combinatorics is	[EC1], §1.1 and online lecture notes
September 20	What combinatorics is (continued), basic combinatorial statistics	[EC1], §1.1 and online lecture notes
September 23	Counting principles: inclusion-exclusion, pigeonhole, twelvefold way (if time permits)	[EC1], §2.1, [Sag20+], §1.8, 2.1
September 24 (x-hour)	Intro to computer programming for math	Online worksheet
September 25	Generating functions, the basics	[Wil94], §1
September 27	Software demo: Guessing	[Sage], Ore-Algebra package, if possible, see online worksheet
September 30	Posets: basic concepts and examples, lattices	[EC1], §3.1, 3.3, [Sag20+], §5.1
October 2	Properties of lattices	[EC1] §3.3, 3.4, [Sag20+], §5.3
October 4	Incidence algebra and Möbius function, part 1	[EC1], §3.6, 3.7, [Sag20+], §5.4, 5.5
October 7	Möbius function, part 2	[EC1], §3.7, 3.8, 3.9, [Sag20+], §5.8
October 9	Brief intro: hyperplane arrangements through posets	[EC1], §3.11
October 11	Software demo: Computing the Möbius function	Online worksheet
October 14	Group acting on sets: intro, Burnside's Lemma	[Sta18], §7, [Sag20+], §6.2
October 16	Group acting on sets: Pólya theory	[Sta18], §7, [Sag20+], §6.4
October 18	Necklaces	[Sta18], §7
October 21	Young tableaux, Symmetric group acting on Young tableaux, bumping and sliding	[Ful97], §1, §7.1
October 23	Robinson-Schensted bijection	[Ful97], §4

Date	Content	References
October 25	Increasing sequences	[Sag01], §3.3, 3.5
October 28	A crash course of representation theory through examples	[Sag01], §1
October 30	Specht modules through tabloids	[Sag01], §2.1, 2.3, 2.4, [Ful97], §7.2
November 1	Basis of Specht modules, dimension	[Sag01], §2.5
November 4	Branching rule for the Symmetric group, induced and restricted representations	[Sag01], §2.8
November 6	The decomposition of a permutation module	[Sag01], §2.9
November 8	The Murnaghan-Nakayama rule	[Sag01], §4.10, [EC2], §7.17
November 11	Student presentations	
November 13	Student presentations	
November 15	Student presentations	
November 18	Student presentations	
November 24 (Sunday)	**Final exam**	

Combinatorics seminar

Dartmouth hosts a combinatorics seminar on some Tuesday, at 11:00. You are highly encouraged to attend it, and I will advertise it in class before each talk. More details can be found on this webpage: `https://math.dartmouth.edu/~comb/`.

Evaluation (tentative)

Upon approval by the students.

The homework assignments will be available at least five days before they are due, and will be due each Wednesday (except for the first homework, that will be due Friday, September 20). There might be one or two problems that you will not be able to solve on the first days the problem set is available, but these will be marked.

Evaluation	Date	Value
Homework	Weekly	45 %
Student presentations or software projects	November 13-18	25 %
Final exam	November 22	30 %

Finding the right answer is not equivalent to solve a problem, and therefore your solution will be evaluated as a whole. Please, keep in mind while writing the assignments that I will not be with you when I will read it, so it must be complete.

The student presentations can be about any research paper in algebraic combinatorics, or chosen in a list of topics that will be given to you mid-October. For those who prefer the software project, your project could consist in implementing an algorithm from algebraic combinatorics into a programming language, writing a tutorial on how to use mathematical software to solve a combinatorial problem or contributing to a free software. Students having further ideas should talk to me about the project they have.

The presentations and projects can be done in teams of two students, but the project should be bigger (longer talk or broader software project).

Textbook

There is **no required textbook** for the class. Instead, all notes are gonna be written on the blackboard and the tentative schedule tells you what reference I'm using on what day. A list of references that I am using to design the course material follows in this syllabus. All books listed here are at the Baker library's reserve, and some of them are freely available online (at least partially). Sagan's *Combinatorics: The Art of Counting* [Sag20+] is not yet published, so you will not find it at the library, but you can access it online for free.

References

[EC1] R. P. Stanley. *Enumerative combinatorics. Volume 1*, volume 49 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge,

second edition, 2012. Freely available online at `http://www-math.mit.edu/~rstan/ec/ec1.pdf`.

[EC2] R. P. Stanley. *Enumerative combinatorics. Volume 2*, volume 62 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1999.

[Ful97] W. Fulton. *Young tableaux. With applications to representation theory and geometry*, volume 35 of *London Mathematical Society Student Texts*. Cambridge University Press, Cambridge, 1997.

[Sag01] B. E. Sagan. *The symmetric group. Representations, combinatorial algorithms, and symmetric functions*. Graduate Texts in Mathematics. Springer-Verlag, New York, second edition edition, 2001.

[Sag20+] B. E. Sagan. *Combinatorics: The Art of Counting*. In preparation. Freely available preliminary version at `https://users.math.msu.edu/users/sagan/Books/Aoc/aoc.pdf`.

[Sage] The Sage Developers. *SageMath, the Sage Mathematics Software System*. `http://www.sagemath.org`.

[Sta18] R. P. Stanley. *Algebraic combinatorics. Walks, trees, tableaux, and more, second edition.* Undergraduate Texts in Mathematics. Springer, Cham, 2018. Freely available without exercices at `http://www-math.mit.edu/~rstan/algcomb/algcomb.pdf`.

[Wil94] H. S. Wilf. *generatingfunctionology*. Academic Press, Inc., Boston, MA, second edition, 1994. Freely available at `https://www.math.upenn.edu/~wilf/gfologyLinked2.pdf`. Only the first edition is available at the library.

Students are encouraged to use software to solve the problems. For combinatorics, SageMath [Sage] is well-developed. It is furthermore free to use, to modify and to share. You can either download it for your personal computer running Windows, MacOS or Linux, or you can use it through CoCalc (`https://cocalc.com/`). The department also has a server running it (`https://math.dartmouth.edu/jupyter`), if you have an account on the Gauss departmental server.

A list of further references for generic skills (note taking, typing mathematics with LaTeX, using SageMath, ...), is available on the website of the course.

Honor Principle

Assignments and examinations serve both the purpose of fixing your ideas on a given subject and, for the evaluator, to assess the comprehension you have of the class and how the class material has been understood. None of these goals can be achieved if you hand out a solution that is taken from someone else or from a publicly available source. Nevertheless, students are allowed to work together on problems to explore their solutions. The homework you submit must reflect your own understanding, and you thus need to write your own copy of the solution, *in your own words*.

For the final exam: Students are not allowed to give or receive any assistance of any form, except from their instructor.

More information on the Academic Honor Principle can be found on Dartmouth's website: `https://students.dartmouth.edu/judicial-affairs/policy/academic-honor-principle`

Dartmouth Policies

Student Accessibility and Accommodations

Students with disabilities who may need disability-related academic adjustments and services for this course are encouraged to see me privately as early in the term as possible. Students requiring disability-related academic adjustments and services must consult the Student Accessibility Services office in Carson Hall 125 or by phone: 646-9900 or email: Student.Accessibility.Services@Dartmouth.edu.
Once SAS has authorized services, students must show the originally signed SAS Services and Consent Form and/or a letter on SAS letterhead to me. As a first step, if you have questions about whether you qualify to receive academic adjustments and services, you should contact the SAS office. All inquiries and discussions will remain confidential.

Mental Health

The academic environment at Dartmouth is challenging, our terms are intensive, and classes are not the only demanding part of your life. There are a number of resources available to you on campus to support your wellness, including your undergraduate

dean (http://www.dartmouth.edu/~upperde/), Counseling and Human Development (http://www.dartmouth.edu/~chd/), and the Student Wellness Center (http://www.dartmouth.edu/~healthed/). Remember, mental and physical health should be your number one priority!

Religious Observances

Some students may wish to take part in religious observances that occur during this academic term. If you have a religious observance that conflicts with your participation in the course, please meet with me as soon as possible, preferably before the end of the second week of the term to discuss appropriate accommodations.

Financial Needs

All material for this class is aimed to be freely available, either at the library or online. If you encounter financial challenges related to this class (because of the workload for example), please let me know.