FINAL EXAM (TAKE-HOME)

ALGEBRAIC COMBINATORICS (MATH 68)

Due November 27, 2019

As this is an exam, **you are not allowed to give or receive any help**, except from the instructor. However, **you are allowed to use the lectures notes** and any assignment you have completed for this course. Other references are not allowed.
You must write the appropriate justification as part of the solutions.

Please, **turn in your solutions by email** (since we won't meet over the period of time for the exam).

(1) (25 points) Prove this interpretation of Erdős-Szekeres theorem: Any permutation of length greater than n^2 must contain either an increasing sequence of length at least n or a decreasing sequence of length at least n. Prove that this bound is sharp (i.e. that there is a permutation of length n^2 with no increasing nor decreasing sequence of length $n+1$).

(2) (25 points) Let A_n denote the alternating subgroup of S_n (i.e. the group of even permutations). Let $\sigma \in S_n$ have cycle type $(\lambda_1, \ldots, \lambda_l)$.
 (a) Show that $\sigma \in A_n$ if and only if $n - l$ is even.
 (b) Explain why A_4 has four irreducible representations.
 (c) Do all characters of A_n have integer values? Why?
 (d) Give two important differences between the table of characters of A_n and of S_n.

(3) (20 points) True or False: If every chain and every antichain of a poset P is finite, then P is finite (as a set). You must justify your answer.

(4) (10 points) How many compositions of 17 use only parts of length 2 and 3?

(5) (20 points) How many distinct regular tetrahedra are there under rotation if the faces are colored from a set with r colors. Also, give a numerical answer for $r = 1, 2, 3, 4, 5$.

Good luck!