

## HOMEWORK V

ALGEBRAIC COMBINATORICS (MATH 68)

Due October 16, 2019, at the **beginning of the class**

Collaboration among students to find key to the solution is encouraged, but each person must write the homework in his/her own words. You must write the name of the students with whom you work for each problem, as well as any written resource (web, book, etc.) that has been extensively used.

You must write the appropriate justification as part of the solutions.

- (1) Consider a poset with a  $\hat{0}$  and a  $\hat{1}$ . Prove that

$$\sum_{s \leq t} \mu(s, t) = 1.$$

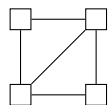
*Hint: You can multiply the left-hand side by  $\delta$ , but  $\delta$  could also be  $f^{-1}f$  for  $f \in \mathcal{I}$ .*

- (2) The *Catalan arrangement* is the set  $\mathcal{C}_n$  of all hyperplanes

$$H_{i,j,k} = \{x_i - x_j = k\}, \text{ for } k \in \{-1, 0, 1\} \text{ and } i, j \in [n],$$

where  $n$  is the dimension of the space.

- (a) How many hyperplanes does  $\mathcal{C}_n$  contains?
  - (b) Draw  $\mathcal{C}_3$ .
  - (c) Draw its intersection poset and compute the Möbius function of each element.
  - (d) Compute the characteristic polynomial of the poset.
- (3) How many regions does the Shi arrangement in dimension 4 have? How many of them are relatively bounded?
- (4) (Only if you want more challenge) Draw the Shi arrangement in dimension 4 (it is 3-dimensional). You can use Sage (try typing 'hyperplane\_arrangements.' and hit 'tab'.)
- (5) Let  $M$  be the matroid associated to the graph below.



- (a) What is its broken circuit complex (if it is too hard to draw, you can write the faces)?
- (b) What is its characteristic polynomial?

**Good luck!**