HOMEWORK IV

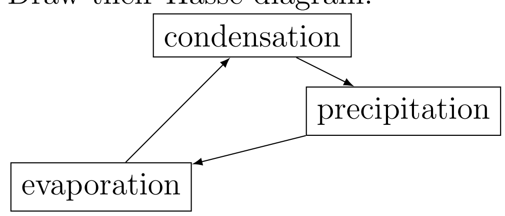
ALGEBRAIC COMBINATORICS (MATH 68)

Due October 9, 2019, at the **beginning of the class**

Collaboration among students to find key to the solution is encouraged, but each person must write the homework in his/her own words. You must write the name of the students with whom you work for each problem, as well as any written resource (web, book, etc.) that has been extensively used.
You must write the appropriate justification as part of the solutions.

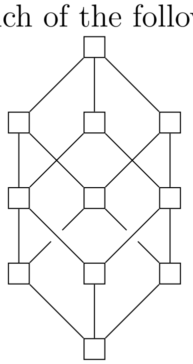
(1) Can you turn the following graphs into posets? Draw their Hasse diagram.

(a)

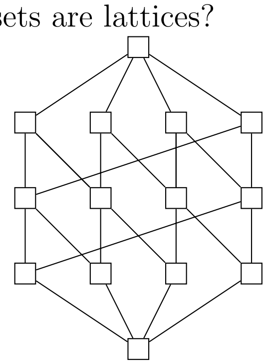


(b)

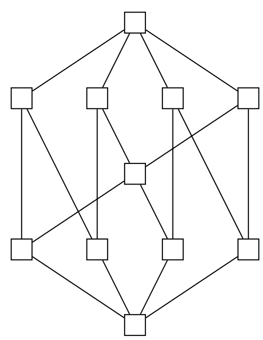
(2) Which of the following posets are lattices?



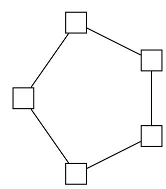
(a)



(b)



(c)



(d)

(3) Show that the Tamari lattice of order 3 is not distributive. Its Hasse diagram is the one from 2d.

(4) What is the rank function for the following posets? I'm expecting you to write a function that maps an item of the poset to its rank without using the Hasse diagram.
   (a) The divisors lattice of n.
   (b) The boolean lattice (subsets of $[n]$).
   (c) The chain of order n ($[n]$).

(5) Prove that the number of maximal chains in the interval $[s, t]$ is given by $(1 - \eta)^{-1}$. (The η function is defined in the lecture notes.)

(6) Let P be an n-element poset, and I an order ideal of P.
   (a) For now, assume $\mathcal{J}(P)$ is ranked. Prove that $\text{rank}(I) = \#I$.
   (b) Prove that $\mathcal{J}(P)$ is ranked of order n.
   (c) Assume that the set of join-irreducible elements of $\mathcal{J}(P)$, considered as an induced subposet of $\mathcal{J}(P)$, is isomorphic to P. Prove that there is a bijection between posets of order n and distributive lattices of rank n.
   Hint: Use FTFDL.

If you have any question, stop by during my office hours (Monday 10-11 and Wednesday 9-11 in Kemeny 229).

Good luck!