

HOMEWORK I

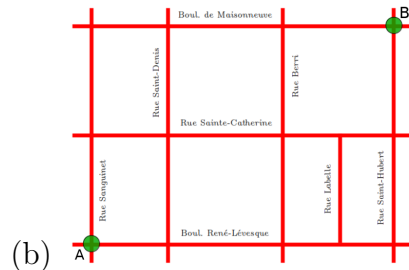
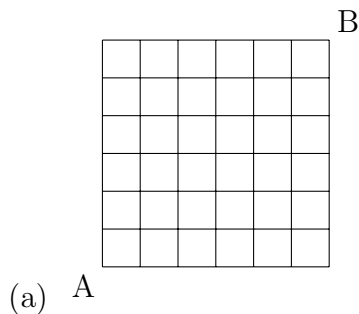
ALGEBRAIC COMBINATORICS (MATH 68)

Due September 20, 2019, at the **beginning of the class**

Collaboration among students to find key to the solution is encouraged, but each person must write the homework in his/her own words. You must write the name of the students with whom you work for each problem, as well as any written resource (web, book, etc.) that has been extensively used.

You must write the appropriate justification as part of the solutions.

- (1) Give the generating function of
 - (a) the permutations of n with exactly n fixed points.
 - (b) the permutations of n with exactly $n - 1$ fixed points.
 - (c) the permutations of n with exactly $n - 2$ fixed points.
 - (d) the Fibonacci numbers (the sequence F such that $F_0 = 0$, $F_1 = 1$ and $F_{n+1} = F_n + F_{n-1}$).
- (2) How many
 - (a) subsets of $\{1, 2, \dots, 10\}$ contain at least one even number?
 - (b) ways of arranging the letters in MISSISSIPPI are there such that the four I's are not contiguous?
 - (c) functions from the set $\{1, 2, 3, 4, 5\}$ to itself are at most two-to-one (i.e. the pre-image of any number is a set of size at most 2)?
- (3) Give a *combinatorial* (i.e. bijective) proof of the following identities:
 - (a) $\sum_{k=0}^n k \binom{n}{k} = n2^{n-1}$.
 - (b) $\sum_{k=0}^n (-1)^k \binom{n}{k} = 0$.
- (4) How many paths from A to B using only North and East steps are there on those grids? Explain your solution.



Don't forget that the solution is not the same as the answer.

Good luck!