Group Work Tip #2. Encourage everybody in the group to participate in the discussion and listen to what the others are saying.

8. Topology of $\mathbb{R}^n$ (continued)

Theorem 8.5. *Let A be a subset of $\mathbb{R}^n$. Show that $\mathbf{x}$ is a limit point of A iff for every $r > 0$ the ball $B_r(\mathbf{x})$ intersects A.*

Exercise 8.6.

> **Definition.** Given a subset A of $\mathbb{R}^n$, the **closure of** A, denoted $\overline{A}$, consists of all the limit points of A.

(a) Prove that A is closed iff $\overline{A} = A$.
(b) Show that $\overline{A}$ is a <u>closed set</u> containing A. (It is in fact <u>the smallest</u> closed set containing A.)

Note. The above Theorem 8.5 gives a convenient method of effectively finding the closure of some sets. Use it to compute the closure of a ball $\overline{B_r(\mathbf{x}_0)}$ in $\mathbb{R}^n$, or to check your answer to 4.3.A in the last homework.

Exercise 8.7.

> **Definition.** The **interior** of a subset A of a metric space X, denoted $\mathrm{int}(A)$, is the largest open set of X contained inside A. Consequently we have
> $$\mathrm{int}(A) \subseteq A \subseteq \overline{A}.$$

If $A \subset \mathbb{R}^n$ has empty interior, must it be closed? (*Hint.* Think at an example.)

Exercise 8.8. Show that a compact subset of $\mathbb{R}^n$ is closed and bounded.

Exercise 8.9. Show that a closed subset C of a compact metric space X is compact.