Exercise 12.5.

(a) Show that the sum of two continuous functions with the same domain is a continuous function.
(b) Show that the product of two scalar valued functions is continuous.
(c) Show that the composition of two continuous functions is a continuous function.

Exercise 12.6.

(a) Consider the **projection functions** $\pi_i : \mathbb{R}^n \to \mathbb{R}$, $\pi_i(\mathbf{x}) = x_i$, where $\mathbf{x} = (x_1, x_2, \ldots, x_n)$, and $i = 1, 2, \ldots, n$. Show that all the projection functions are continuous. What is the philosophical reason behind this result?
(b) Every function $f : S \subset \mathbb{R}^n \to \mathbb{R}^m$ can be written as $f(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_n(\mathbf{x}))$, where $f_i = \pi_i \circ f$ is the i^{th} **coordinate of** f. Show that f is continuous if and only if all the f_i's are continuous.

Exercise 12.7. Consider metric spaces X and Y, and a continuous function $f : X \to Y$. Show that for every compact subset C of X, its image under f, $f(C)$, is compact in Y.

Exercise 12.8. Consider metric spaces X and Y, and a continuous function $f : X \to Y$. If C is compact in Y, must $f^{-1}(C)$ be compact?