Exercise 9.5. Can you give examples of convergent series for which any rearrangement has the same sum?

Exercise 9.6. Prove the following:

> **Theorem.** *For an absolutely convergent series, every rearrangement converges to the same sum.*

(*Hint.* Recall the Cauchy's criterion for series.)

Exercise 9.7. Consider the *conditionally convergent* series $\sum_{n=1}^{\infty} a_n$. Denote by $b_1, b_2, \ldots, b_n, \ldots$ the positive terms of the series and by $c_1, c_2, \ldots, c_n, \ldots$ the negative terms. Show that $\sum_{n=1}^{\infty} b_n$ and $\sum_{n=1}^{\infty} |c_n|$ both diverge. In particular this implies that there are infinitely many positive and negative terms in a conditionally convergent series.

Exercise 9.8. (**Rearrangement Theorem**) Prove that if $\sum_{n=1}^{\infty} a_n$ is a conditionally convergent series, then for every real number L, there is a rearrangement that converges to L.

Exercise 9.9. Let $\sum_{n=1}^{\infty} a_n$ be a series of real numbers. Assume that there exists M such that:

$$\Big| \sum_{n \in F} a_n \Big| \leq M, \text{ for every finite } F \subset \mathbb{N}^*.$$

Show first that $\sum_{n \in F} |a_n| \leq 2M$, for every finite $F \subset \mathbb{N}^*$. Conclude that the series is absolutely convergent.

Exercise 9.10. If a series has every rearrangement convergent, is it true that the series is absolutely convergent?