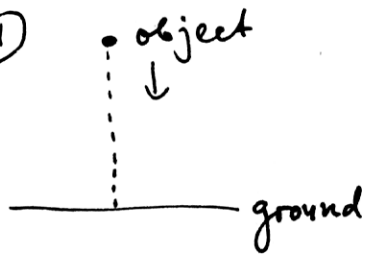
Solutions to the Math Exercises on pages 237/238 in your textbook

① 

in general, $\text{average velocity} = \dfrac{16(t+h)^2 - 16t^2}{h} = 32t + 16h$

time intervals	[5, 5.5]	[5, 5.4]	[5, 5.3]	[5, 5.2]	[5, 5.1]
t	5	5	5	5	5
h	0.5	0.4	0.3	0.2	0.1
average velocity = $= 32t + 16h$	168	166.4	164.8	163.2	161.6

$\Rightarrow$ The value of the velocity at 5 seconds is 160 ft/sec.

② as in ①, we'll put the results in a table.

time interval	[9, 9.1]	[9, 9.01]	[9, 9.001]	[9, 9.0001]	[9, 9.00001]
t	9	9	9	9	9
h	0.1	0.01	0.001	0.0001	0.00001
average velocity $= 32t + 16h$	289.6	288.16	288.016	288.0016	288.00016

$\Rightarrow$ the value of the velocity at 9 seconds is 288 ft/sec.

(3) $7 + h \xrightarrow[h \to 0]{} 7$

i.e., $\lim_{h \to 0} (7 + h) = 7$

$\Rightarrow$ the numbers of the form $7+h$ approach 7 as $h \to 0$.

(4) $\dfrac{(4+h)^2 - 4^2}{h} = \dfrac{4^2 + h^2 + 8h - 4^2}{h} = h + 8$

(If you prefer, you can use the formula $a^2 - b^2 = (a-b)(a+b)$ to simplify $(4+h)^2 - 4^2$, namely $(4+h)^2 - 4^2 = (4+h-4)(4+h+4) = h(h+8)$)

⑤ $[4, 4+h] \Rightarrow t=4$

$$\text{average velocity} = \frac{16(4+h)^2 - 16\cdot 4^2}{h} = 16\underbrace{\left(\frac{(4+h)^2-4^2}{h}\right)}_{\text{④}}$$

④ $= h+8$

$\Rightarrow$ average velocity $= 16(h+8) = 16h+128$

⑥ as seen in ④,

$$\frac{(4+h)^2-4^2}{h} = h+8 \xrightarrow[h\to 0]{} 8$$

$$\Rightarrow \lim_{h\to 0} \frac{(4+h)^2-4^2}{h} = 8$$

i.e., $\frac{(4+h)^2-4^2}{h}$ approaches 8 as $h \to 0$.

⑦ average velocity $= 32t+16h$

when $t=4 \Rightarrow$ average velocity $= 32\cdot 4 + 16h = 16h+128$

$$\text{instantaneous speed}\Big|_{t=4} = \lim_{h\to 0}(16h+128) = 128$$

i.e., the velocity at 4 seconds is 128 ft/sec.

(8) $s(t) = 4.9t^2$ (meters)

$[t, t+h]$

$\Rightarrow$ average velocity $= \frac{4.9(t+h)^2 - 4.9t^2}{h} =$

$= 4.9\left(\frac{(t+h)^2 - t^2}{h}\right) = 4.9\ (2t+h)$

$\Rightarrow$ average velocity $= 9.8t + 4.9h$ m/sec.

(9) $\lim_{h \to 0} (9.8t + 4.9h) = 9.8t$ m/sec.

the above limit represents the instantaneous speed at t.

(10) $v(t) = 9.8t$ m/sec

$[t, t+h]$

$\Rightarrow$ average acceleration $= \frac{9.8(t+h) - 9.8t}{h} = \frac{9.8t + 9.8h - 9.8t}{h}$

$\Rightarrow$ average acceleration $= 9.8$ m/sec^2

$\Rightarrow$ acceleration at time t is 9.8 m/sec^2

(i.e., the acceleration is constant)