Solutions to the Math Exercises on pages 159/160 in your textbook

① A •————————• B

the distance between the points A and B is equal to the absolute value of the arithmetic difference of the numbers A and B, i.e., $|A-B|$

the set of points on the line AB is the set of all real numbers between A and B.

$\Longrightarrow$ the distance between the points A and B is equal to the length of the interval $[A, B]$ which is different than the set of all real numbers between A and B – the latter is infinite. (compare w/ pages 151/153)

if $A=1$, $B=5$,

then the distance between A and B is equal to $|1-5|=4$;

there are infinitely many points from 1 to 5.

②

n	2	3	4	5
$\left(\frac{1}{2}\right)^{n+1}$	$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$	$\left(\frac{1}{2}\right)^4 = \frac{1}{16}$	$\left(\frac{1}{2}\right)^5 = \frac{1}{32}$	$\left(\frac{1}{2}\right)^6 = \frac{1}{64}$

$$\lim_{n\to\infty}\left(\frac{1}{2}\right)^{n+1} = \lim_{n\to\infty}\frac{1}{2^{n+1}} = \frac{1}{\lim_{n\to\infty} 2^{n+1}} = 0$$

$$\Rightarrow \lim_{n\to\infty}\left(\frac{1}{2}\right)^{n+1} = 0$$

③

n	2	3	4	5
$\left(\frac{3}{2}\right)^{n+1}$	$\left(\frac{3}{2}\right)^{3} = 3.375$	$\left(\frac{3}{2}\right)^{4} = 5.0625$	$\left(\frac{3}{2}\right)^{5} = 7.59375$	$\left(\frac{3}{2}\right)^{6} = 11.390625$

as $n \to \infty$, $\left(\frac{3}{2}\right)^{n+1}$ get large w/o bound

(any time we multiply by $\frac{3}{2} = 1.5 > 1$ and start w/ 1.5)

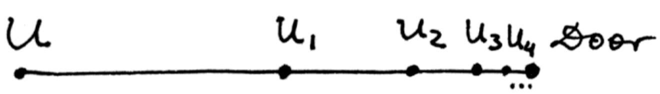
④ $1 + r + r^2 + \cdots + r^n + \cdots = \dfrac{1}{1-r}$ for $0 < r < 1$

$\Rightarrow$ for $r = \frac{9}{10} \in (0,1)$

have

$$1 + \frac{9}{10} + \left(\frac{9}{10}\right)^2 + \left(\frac{9}{10}\right)^3 + \cdots = \frac{1}{1-\frac{9}{10}} = \frac{1}{\frac{1}{10}} = 10$$

⑤



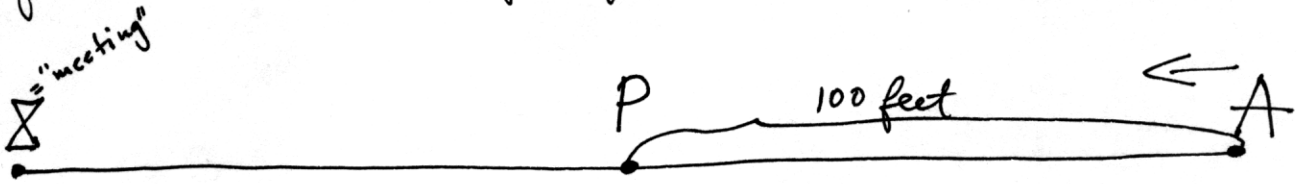
Zeno would say that u never reach the Door since in order to do that, u first have to go half way to the Door and so on ... which is impossible in <u>his world</u>

A mathematician would get to the Door :

(otherwise they can <u>never</u> go to their lectures - we all know from experience that mathematicians show up for their classes, in other words - they make it to the door and through many other doors during the process - this is both a joke and for real)

Indeed, it is possible to pass through an infinite number of points in a finite time (p 156)

6



Imagine that the above line is straight.

A denotes the archer w/ the arrow ;

P denotes the prisoner ;

One of your favorite formulas says :

$$\boxed{\text{distance} = \text{speed} \cdot \text{time}}$$

let the arrow and the prisoner "meet" in X seconds from the time the arrow is released and the prisoner starts running
(I don't like cruelties ⇒ I use "meet")

Since distance = speed · time

$$\Rightarrow \quad speed = \frac{distance}{time}$$

let V_P denote the speed of the prisoner ;

let V_A denote the speed of the arrow ;

By , $V_P = \dfrac{50}{\frac{1}{2}} = 100$ feet/s

$$V_A = \frac{100}{\frac{1}{2}} = 200 \text{ feet/s}$$

Now, the distance which the prisoner runs for x seconds is equal to $(100 \cdot x)$ feet, and the distance which the arrow travels for x seconds is equal to $(200 \cdot x)$ feet

Since the initial distance between the arrow and the prisoner is 100 feet, the relationship is :

$$100 + 100x = 200x$$

$$\Rightarrow 100 = 200x - 100x$$

$100 = 100x \Rightarrow x = 1 \Rightarrow$ it takes 1 second for the arrow and the prisoner to "meet" after the moment of the release of the arrow

(a) the arrow travels $200 \cdot 1 = 200$ feet in 1 second

(b) the prisoner moves $100 \cdot 1 = 100$ feet in 1 second

(c) the prisoner escapes his meeting with the arrow because the arrow is from Zeno's quiver (his wish!)