Solutions to the Math Exercises on pages <u>33 / 34</u> in your textbook

① a wave is traveling at a speed $v = 3 \cdot 10^8$ m/s with a frequency $f = 4$ Hertz (cycles/s).

Denote the wavelength with <u>λ</u> and use the formula on page 19, namely:

$$\boxed{\text{wavelength} = \frac{\text{velocity}}{\text{frequency}}},$$

to compute

$$\lambda = \frac{3 \cdot 10^8}{4} \ \text{m} = \frac{3 \cdot 10^6 \cdot 100}{4} = \frac{3 \cdot 10^6 \cdot 25}{1} = 75 \cdot 10^6 \ \text{m}$$

② the speed of a wave is $v = 2$ m/s and its wavelength $\lambda = 4$ <u>cm</u>

Since the wavelength is given in <u>cm</u>, first we compute it in <u>meters</u> in order to have a <u>consistent set of units</u> : $\lambda = 4 \ \text{cm} = \frac{4}{100} \ \text{m}$.

Now we are ready to compute the frequency, using the above formula: (express the frequency)

$$\boxed{\text{frequency} = \frac{\text{velocity}}{\text{wavelength}}}$$

Denote the frequency with f

and then $f = \dfrac{2}{\frac{4}{100}} = 2 \cdot \frac{100}{4} = 50 \text{ H } (c/s)$

(3) a wave has a wavelength $\lambda = 10$ cm

and frequency $f = 40$ c/s

In order to compute the speed v,

we need the wavelength in meters :

$$\lambda = 10 \text{ cm} = \frac{10}{100} \text{ m} = \frac{1}{10} \text{ m}$$

and the formula $\boxed{\text{velocity} = \text{wavelength} \cdot \text{frequency}}$

$$\Longrightarrow v = \frac{1}{10} \cdot 40 = 4 \text{ m/s}$$

(4) Denote the speed of the first wave with v_1
and the speed of the second wave with v_2 ;
the frequency of the first wave f_1
and the frequency of the second wave f_2 ;
let the wavelength of the first wave be λ_1
and the wavelength of the second wave be λ_2

we know that :

$V_1 = V_2 = 60 \text{ m/s}$

$f_1 = 5f_2$

we want to find a relationship between λ_1 and λ_2

By the formula, $\boxed{\text{wavelength} = \frac{\text{velocity}}{\text{frequency}}}$,

applied for the first and the second wave :

$$\lambda_1 = \frac{60}{f_1} = \frac{60}{5f_2} \quad (\text{since } f_1 = 5f_2)$$

$$\lambda_2 = \frac{60}{f_2}$$

Now compute $\dfrac{\lambda_1}{\lambda_2} = \dfrac{\frac{60}{5f_2}}{\frac{60}{f_2}} = \dfrac{60}{5f_2} \cdot \dfrac{f_2}{60} = \dfrac{1}{5}$

so, $\dfrac{\lambda_1}{\lambda_2} = \dfrac{1}{5}$

which means that $\lambda_1 = \frac{\lambda_2}{5}$ (or $\lambda_2 = 5\lambda_1$)

i.e., the wavelength of the second wave is 5 times the wavelength of the first wave.

Note:
the solution can also be found by using the fact that $v_1 = v_2$ and $f_1 = 5f_2$,

namely: $\lambda_1 = \frac{v_1}{f_1} = \frac{v_2}{5f_2} = \frac{1}{5} \cdot \frac{v_2}{f_2} = \frac{1}{5}\lambda_2$

i.e., $\lambda_1 = \frac{1}{5}\lambda_2$

(5) Again, denote the speed of the first wave with v_1, the speed of the second wave with v_2,

the frequency of the first wave with f_1,
the frequency of the second wave with f_2,
the wavelength of the first wave with λ_1,
the wavelength of the second wave with λ_2.

we know that:

$v_1 = v_2$

$\lambda_1 = 12 \text{ cm} = \frac{12}{100}$ ~~m~~

$\lambda_2 = 36 \text{ cm} = \frac{36}{100} \text{ m}$

$f_2 = 19 \text{ c/s}$

want to find $f_1 = ?$

Your favorite formula says: $\boxed{\text{velocity} = \text{wavelength} \cdot \text{frequency}}$

For the first wave, $v_1 = \lambda_1 \cdot f_1 = \frac{12}{100} \cdot f_1$

For the second wave, $v_2 = \lambda_2 \cdot f_2 = \frac{36}{100} \cdot 19$

But $v_1 = v_2$, i.e., the left-hand sides are equal in , then the right-hand sides must be equal too:

$$\frac{12}{100} \cdot f_1 = \frac{36}{100} \cdot 19$$

$$\Rightarrow f_1 = 3 \cdot 19 = 57 \quad \# (c/s)$$

6

$$\log(2^0) = ?$$

the formula $\log(2^t) = t$ for $t = 0$ gives

$$\log(2^0) = 0$$

another way to get the result is:

$$\log(2^0) \underset{\substack{\uparrow \\ 2^0 = 1}}{=} \log(1) = 0$$

7. $2^2=4 \Rightarrow \log(4)=2$

$2^3=8 \Rightarrow \log(8)=3$

$2^4=16 \Rightarrow \log(16)=4$

$2^5=32 \Rightarrow \log(32)=5$

8. $$\log\left(8^{\frac{1}{3}}\right) \underset{\substack{\uparrow \\ 8=2^3}}{=} \log\left(2^{3\cdot\frac{1}{3}}\right) = \log(2) = 1$$

$$\log(64) \underset{\substack{\uparrow \\ 64=2^6}}{=} \log(2^6) = 6\log(2) = 6$$

$$\log(\sqrt{2}) = \log\left(2^{\frac{1}{2}}\right) = \frac{1}{2}\log(2) = \frac{1}{2}$$

9. $$\log\left(\frac{2^8}{8^2}\right) = \log(2^8) - \log(8^2) =$$

$$= 8\log(2) - \log(2^{3\cdot 2}) =$$

$$= 8 - \log(2^6) =$$

$$= 8 - 6\log(2) = 8 - 6 = 2$$

or you can do it as follows:

$$\log\left(\frac{2^8}{8^2}\right) = \log\left(\frac{2^8}{2^{3\cdot 2}}\right) = \log\left(\frac{2^8}{2^6}\right) =$$

$$= \log(2^2) = 2\log(2) = 2$$

(10) $$\log\left(\frac{128}{2^3}\right) = \log(128) - \log(2^3) =$$

$$= \log(2^7) - \log(2^3) =$$

$$= 7\log(2) - 3\log(2) = 7 - 3 = 4$$

or

$$\log\left(\frac{128}{2^3}\right) = \log\left(\frac{2^7}{2^3}\right) = \log(2^4) =$$

$$= 4\log(2) = 4.$$

(11) $$\log\left(\frac{4}{3}\right) - \log\left(\frac{3}{4}\right) =$$

$$= \log(4) - \log(3) - \left(\log(3) - \log(4)\right) =$$

$$= \log(2^2) - \log(3) - \log(3) + \log(2^2) =$$

$$= 2\log(2) - \log(3) - \log(3) + 2\log(2) =$$

$$= 4\log(2) - 2\log(3) =$$

$$= 4 - 2\log(3)$$

or,

$$\log\left(\frac{4}{3}\right) - \log\left(\frac{3}{4}\right) =$$

$$= \log\left(\frac{4}{3}\right) + \log\left(\left(\frac{3}{4}\right)^{-1}\right) =$$

$$= \log\left(\frac{4}{3}\right) + \log\left(\frac{4}{3}\right) =$$

$$= 2\log\left(\frac{4}{3}\right) = 2\left(\log(4) - \log(3)\right) =$$

$$= 2\left(\log(2^2) - \log(3)\right) =$$

$$= 2\left(2\log(2) - \log(3)\right) =$$

$$= 2\left(2 - \log(3)\right) = 4 - 2\log(3)$$

12 Your calculators probably have $\log_{10}$

and $\ln = \log_e$

Use the <u>change of base formula</u>:

$$\log_2 x = \frac{\log_{10} x}{\log_{10} 2} \quad \text{or} \quad \log_2 x = \frac{\log_e x}{\log_e 2} = \frac{\ln(x)}{\ln(2)}$$

for (a), $x = \frac{9}{8}$

for (b), $x = \frac{2^8}{3^5}$

the answer is: (b) < (c) < (a)

((b) = 0.0752

(c) = 0.0833

(a) = 0.085)

so, the length of a Pythagorean half-step $\left(\log \frac{2^8}{3^5}\right)$ is the smallest,

followed by the length of a well-tempered half-step $\left(\frac{1}{12}\right)$ and then

by the length of half a Pythagorean whole-step $\left(\frac{1}{2} \log \frac{9}{8}\right)$