Critique of Paper Two: Who was Publius?

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This narrative is well organized and relates material that is both informative and engaging. I enjoyed reading it, despite the distracting composition problems I have noted.

During the framing of the U. S. Constitution, a series of articles called *The Federalist Papers* was published in New York under the pseudonym Publius. Written by Alexander Hamilton, James Madison and John Jay, the purpose of *The Federalist Papers* was to encourage the voters of New York to support the new federal Constitution. In all, 85 papers were written and published in newspapers later to be compiled into book form. Historians agree that Jay wrote 5, Madison wrote 14, and Hamilton wrote 51 (p. 3). This leaves 15 papers unaccounted for. Three of them are attributed to Madison and Hamilton together while the remaining twelve are disputed.

In 1964, mathematicians Frederick Mosteller and David Wallace conducted a statistical study of *The Federalist Papers* in an attempt to determine the author of the disputed papers. This paper will attempt to summarize their efforts and show how mathematical principles can be applied to other fields of knowledge such as history. Obviously the math is greatly simplified and only the main theories are presented. Statistics can be manipulated in numerous and complex ways beyond the scope of this paper.

The authorship of these disputed papers has been a hotly debated topic among historians. Several lists have been uncovered but they are contradictory and appear to be full of errors. Madison claimed authorship for all of the disputed papers many years after they were originally published, however the contradictory lists caused doubt about his

claim. Mosteller and Wallace have taken a scientific approach to the problem and attempted to solve it by addressing the text itself. They postulated that through a statistical analysis of the text they could discern the author from the writing style. Historians have not been able to do this because Hamilton and Madison had very similar styles of writing, but Mosteller and Wallace decided to look at the frequency of word use.

Being humble mathematicians and not wanting to upset haughty historians, Mosteller and Wallace made use of a rarely used method of statistical analysis known as Bayes' Theorem. This allowed them to combine initial odds of authorship determined by historical evidence with the likelihood ratio of authorship determined by their study of words. This is the basis for Bayes' Theorem where p1 = the initial or prior probability of hypothesis 1 and p2 is the prior for hypothesis 2. Let p2 = 1 - p1. Let f1(x) = the conditional probability of hypothesis 1, that is hypothesis 1 as determined by the experiment, and f2(x) = the conditional probability of hypothesis 2. Bayes' Theorem says that the final odds of hypothesis 1 with respect to hypothesis 2 are equal to the initial odds times the likelihood ratio determined by the experiment.

Odds (hyp1, hyp2) = (p1/p2) (f1(x)/f2(x)). (p. 54)

This takes into equal account the prior hypothesis and the conditional probability. Therefore if historical evidence favored one hypothesis strongly, the experimental results would not change that, just reinforce it or weaken it, but if historical evidence was ambiguous, the results would have a much stronger effect.

To begin their study, Mosteller and Wallace had to choose a discriminating factor. Since Hamilton and Madison had such similar writing styles, words were the best choice. They used certain words at different rates. By examining papers known to be written by

Hamilton and Madison, they were able to come up with a pool of words to examine in the disputed papers. They selected for words which appeared more frequently in papers by one of the authors. All words having to do with the context of the paper were eliminated from the count to avoid context clouding the author's style of writing. Through several screening processes, 30 words were chosen to conduct the analysis of the disputed papers. These included both high and low frequency words, but were selected for their discriminating ability and their objectivity with regard to context. Each word was counted in the known papers and assigned a value μ_H for the rate of occurrence per thousand words in Hamilton's papers and μ_M in Madison's papers. These words were then counted in papers of known authorship which had not been used in selecting these words and analyzed as a control along with the study of the disputed papers.

With the values of μ_H and μM determined for the 30 words, it was quite easy to assign each word a weighting as a discriminating factor. The likelihood ratio for a group of words was the product of the probabilities determined by the μ's

PμH/PμM *PμH/PμM. (p. 54)

If μH was high and μM was low or the other way around, the weighting was strong in favor of either Hamilton or Madison, but if the μH and μM were similar than the fraction would be almost equal to 1 and the word would not carry much weight in determining authorship. The likelihood ratio of Hamilton to Madison for x is

$$K = (\mu_H/\mu_M)_x e_{-\omega(\mu}$$

H

μ

M

) (p. 55)

Where ω = the number of words in thousands and e is the natural logarithmic base. By this formula, when it is more likely that Hamilton wrote a paper, the likelihood ratio is greater than 1 and when it is more likely that Madison wrote a paper, it is less that 1. Because of this, the likelihood ratios are more easily analyzed in logarithmic form where a positive log likelihood ratio implies Hamilton and a negative implies Madison. The log likelihood ratio is obtained by taking the log of each side and is

$$\lambda(x) = x\log(\mu_H/\mu_M) - \omega(\mu_H - \mu_M). \text{ (p. 55)}$$

A combination of this log likelihood ratio with the prior ratio by Bayes' Theorem yielded the final log odds which were used to determine authorship (positive – Hamilton, negative – Madison). All of the prior odds were assumed by Mosteller and Wallace to be 1-1 allowing historians to insert their own odds if they had convincing evidence one way or the other for an individual paper. They hoped that unless the prior odds were very strong, the log likelihood ratio would overwhelm them and give a convincing determination of authorship.

The main study was conducted by (at the time) high speed computers. The word counts were recorded on punch cards and the most expensive part of the whole undertaking was getting the text of the papers into the computer. As a control, the log odds were computed for all of the known federalist papers by both authors and they showed convincingly that the method worked. A log odd of 0 work mean that the study was unable to determine anything and a log odd of over 15 (either positive or negative) meant that the odds were over 1 million to one in favor of that author. Most of the log odds for the known papers were around positive or negative 15. (pp. 70-73) Once they were satisfied that the method worked for the control, the disputed

papers were analyzed. The 12 disputed papers and 3 joint papers all yielded negative log odds indicating that they all had been written by Madison. Even more surprising was that nearly all of the disputed paper had log odds of –15 or less yielding odds of over 1 million to 1 in favor of Madison. (p. 74) This was truly a major historical breakthrough. Very few things can be said to have happened with a million to 1 certainty. Of course Mosteller and Wallace did not take this at face value. Most of their book is in fact dedicated to the analysis of different sources of error. They even conducted three parallel studies using different methods to check their results and remained convinced of their validity.

This analysis has convincingly solved a long debated historical question. It is a perfect example of how math can be utilized in other fields of study. What made this study so unique was its use of Bayes' Theorem which allowed a merging of fields. Instead of a purely quantitative statistical analysis based on word counts, qualitative historical evidence could be taken into consideration. This approach could be very useful in other questions of disputed authorship and not just in the field of history. Perhaps someone could undertake an analysis of Shakespeare's play and determine whether he really wrote all of them. It could also have a role in the field of law perhaps determining similar questions in the courtroom. It is really incredible that a statistical analysis of words could make such a powerful determination of authorship.

References

Mosteller, Frederick and David Wallace. *Inferred and Disputed Authorship: The Federalist.* Reading: Addison-Wesley, 1964.

The Good

Narrative is well organized.

Topic is discussed in detail and supported with examples of a well-defined mathmatical process.

Topic is original, and the hint of mystery makes the paper appealing to readers other than mathematicians.

The Bad

Let's skip this category and go directly to

The Ugly

This paper contains a wealth of errors in punctuation, usage, word choice, and sentence structure. This is a shame, because the paper is rich with interesting information, but the reader must fight to follow.