Our "Coin" and the Language of Hypothesis Testing

Math 5 Crew

Department of Mathematics

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The Null Hypothesis

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- Our Example: "Heads" and "tails" are both equally likely.

The Alternate Hypothesis

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- Our Example: Our "coin" is biased and either "heads" or "tails" is more likely.

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- Our Example: Number of "heads".

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- **Before** performing your test you must determine the values of this test statistic for which you will *accept* the null hypothesis and the values for which you will *reject the Null Hypothesis* and *accept the Alternate Hypothesis*.

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- Our Example: We will accept if 7 < Number Heads < 18, and reject otherwise. Hence, the critical region is the collection of integers K that satisfy either $0 \leq K \leq 7$ or $18 \leq K \leq 25$.

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- Our Example: Should be near 5 percent. Check it! .

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- The risk of a type 1 error is called the *Significance Level* of the experiment.
- In order to assure yourself that you can call your results *statistically significant*, you must set you significance level to be less than 5 percent.
- In order to assure yourself that you can call your results *highly significant*, you must set you significance level to be less than 1 percent.

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- Discussion: How might you approximate the risk of a type 2 error in our setting?

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- Relative to an assumption about the statistic's behavior under the alternate hypothesis, the *power* of a hypothesis test is the probability that you correctly accept the alternate hypothesis under this assumption.
- Our Example: Assume the "coin" has a 40 percent chance of coming up "heads". What is the power of our test?

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- Determine:
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- **and** the test's protocol. For example, can you make it double blind? Are there any obvious confounding factors? What equipment and how much time will you need?